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Modal Analysis of a Quad-Rotor Wind Turbine

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Abstract. Unlike Single-Rotor wind turbines, stability analysis of Multi-Rotor wind turbines is still in its initial stages. This paper presents the modal analysis of a Quad-Rotor wind turbine and identifies the new modes or possible instability modes that are otherwise not present on a Single-Rotor wind turbine. Multi-Blade Coordinate transformation scheme is adapted to a Quad-Rotor wind turbine to write the system's equation of motion in fixed-reference frame followed by Eigenvalue analysis to determine the natural frequencies and mode shapes of the Quad-Rotor wind turbine. A Campbell diagram of the Quad-Rotor wind turbine is presented. Results indicate that the Quad-Rotor turbine is soft-soft to the first tower modes (fore-aft, side-side, and torsion). Furthermore, the modes with low natural frequency other than the tower modes are a combination of tower, boom, and blade modes. Therefore, due to the presence of blade modes, the modal frequency of these modes increases or decreases with increasing rotor speed due to centrifugal stiffening.

Keywords: Multi-Rotors, Eigenvalue, Mode shapes, Modal analysis, Campbell diagram

1. Introduction

The key metric in the wind energy industry is the Cost of Energy (COE) which is critical in measuring the success of generating electricity from wind compared to other energy sources. This prompts both the scientific community and the industry to continuously search for ways to reduce the COE in order to make it competitive with already established methods for generating electricity. One way of reducing the COE is by increasing the Annual Energy Production (AEP). AEP is increased by expanding the rotor swept area in order to maximize energy capture from wind. As a result, the trend in wind energy industry is towards larger wind turbines, especially for offshore wind farms. Unfortunately, scaling of the existing wind turbine components increases the COE since AEP is proportional to the square of rotor radius while the cost of wind turbine components is proportional to the cubic of rotor radius, occasionally referred to as the square-cube law in the wind energy community. An alternative to the square-cube law is the Multi-Rotor concept, where two or more rotors are placed on a single tower.

Multi-Rotor wind turbines are not a new concept. The concept dates back to the early 19th century, where a Danish wind mill is modified to a twin-rotor wind mill [1]. The first Multi-Rotor wind turbine concept for generating electric power is proposed in 1932 [2]. The Dutch company Lagerwey, in the late 20th century, built and operated several Multi-Rotor wind turbines [3]. The proposed concepts are comprised of two, four and six two-bladed rotors on a single tower. NASA laboratory tested in 2010 a Multi-Rotor wind turbine consisting of seven rotors [4]. Recently in 2016, Vestas Wind Energy Systems A/S constructed and tested four three-bladed V29-225KW rotors on a single tower [5]. The experiment lasted for about three years and decommissioned in late 2018.

Great progress is being made with regards to the aerodynamic performance of Multi-Rotor wind turbines. Several aerodynamic research endeavours on Multi-Rotor wind turbines have shown an increase in energy extraction compared to a Single-Rotor wind turbine with equal swept area [6, 7]. The distance between rotors on a Multi-Rotor wind turbine influences the aerodynamic interaction of the rotors [6, 8]. Furthermore, it is observed that closely spacing the rotors on a Multi-Rotor wind turbine increases the loading on the blades which requires a further examination by means of aeroelastic simulation of a Multi-Rotor wind turbine [6]. An aerodynamic analysis of a Multi-Rotor wind turbine comprised of four rotors indicate faster wake recovery as well as reduced Turbulence Kinetic Energy

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(TKE) compared to a Single-Rotor wind turbine with equal swept area [9–11]. The reduction of TKE in Multi-Rotor wind turbine wake has the potential of reduced fatigue loads on the downstream wind turbines in a wind farm.

Multi-Rotor wind turbines retain the economic advantage of smaller scale systems while achieving the power gain of larger Single-Rotor wind turbines with equivalent swept area. The ratio of the (rotor) mass of Multi-Rotor turbine to that of a Single-Rotor system with equal swept area is $\frac{1}{\sqrt{n}}$, where n is the number of rotors on the Multi-Rotor system [4]. This ratio indicates that the rotor mass of a Multi-Rotor wind turbine decreases with increased number of rotors, compared to a Single-Rotor wind turbine with equal swept area. However, it does not account for the added mass of the support structure (booms that connect the small rotors to the tower) and possible mass increase of the tower as a consequence of altered loading on the tower compared to a tower with a Single-Rotor. A structural analysis is carried out for a Multi-Rotor wind turbine consisting of $45 \times 444 \text{ kW}$ rotors on a single tower, designed for offshore environment [8, 12]. The design considered pertinent loads cases from IEC-61400-1 [13]. The findings suggest that the benefit in COE reduction for Multi-Rotor turbine due to reduced rotor and drive train cost will not be significantly affected by the increased cost of the support structure on a Multi-Rotor system. Furthermore, Multi-Rotor wind turbines have additional advantage over Single-Rotor wind turbine, such as quicker response time of the smaller rotors to variation in wind speed and standardization of components that increases the reliability thus drive down the cost of energy. However, the model used in [8, 12] does not account for the aeroelastic behaviour of the Multi-Rotor wind turbine, making its findings preliminary at best. Additionally, the aerodynamic model used in [8, 12] does not account for the effect of tower and support structure on the aerodynamic loads. An aeroelastic model is presented for a Multi-Rotor wind turbine consisting of two rotors [14]. The paper concludes that the torsional stiffness of the tower for a Multi-Rotor system is more crucial than for a Single-Rotor case. The work is still ongoing and several improvement points are proposed by the authors to modify the aeroelastic model of the Multi-Rotor wind turbine.

Although extensive research is underway with regards to the aerodynamics of Multi-Rotor wind turbines, there is still work to be done on the complete system design of Multi-Rotor systems. Accurate aeroelastic model of a Multi-Rotor wind turbine needs to be formulated in order to analyse the aeroelastic response of Multi-Rotor systems under turbulent wind loading and examine the aeroelastic stability of a Multi-Rotor wind turbine [15]. Unlike Single-Rotor wind turbines, stability analysis of Multi-Rotor turbines is still in its initial stages. Among the few research endeavours on stability analysis of Multi-Rotor wind turbines, an approach for modal response of two three-bladed rotors is presented in [16]. *This paper presents a modal analysis of a Quad-Rotor wind turbine and identifies new modes or possible instability modes that are otherwise not present on a Single-Rotor wind turbine.*

2. Model Description

The Quad-Rotor wind turbine is modelled using the multi-body simulation (MBS) software SIMPACK [17] coupled with NREL aerodynamics software AerodynV15 [18] to calculate the aerodynamic loads on wind turbine blades. SIMPACK is an MBS software that employs both rigid and flexible bodies to analyse nonlinear response of multi-body systems.

Figure 1b shows the SIMPACK model for the quad-rotor wind turbine; it consists of four WindPact 1.5MW rotors [19]. The blades, booms (parts that connect the nacelle with the tower), and the tower are modelled as flexible bodies, where modal approach is used to model the deformation of the flexible bodies. The rest of the structure (nacelle, hub, etc..) are modelled as rigid bodies. Figure 1a shows the structural layout and dimensions of the tower-boom assembly for the Quad-Rotor turbine. The tower is 136.75m long with four 36.75m long booms attached to the tower in Double-T configuration with the top and bottom booms separated by 73.5m. The tower is a hollow cylinder made of steel with Young's modulus of $2.1 \times 10^{11} \text{ N/m}^2$ and mass density of 8100 kg/m^3 . The bottom 70% of the tower consists of

a constant outer diameter of 4.3m and tapers down to 3.7m at the tower top. The elastic deformation of the tower is modelled using mode shapes corresponding to the first two Fore-Aft (FA) and Side-Side (SS) modes together with the first torsion (TR) mode.

The booms are $0.5(1 + \delta_{sep})D$ long, where D is the rotor diameter and δ_{sep} is the minimum rotor separation distance, set to 5% of rotor diameter in the current model [5]. The booms are made of the same material as the tower and are modelled as a tapered cylinder with the maximum diameter of 3.4m realized at the intersection between the booms and the tower (see Figure 1a). The elastic deformation of the booms is modelled using linear combination of the boom's first two Fore-Aft and Up-Down (UD) mode shapes together with the torsion mode. The fore-aft and up-down mode shapes describe respectively the out-of-plane and in-plane deformation of the booms, with in-plane defined as the plane containing the tower-boom assembly. The elastic deformation of the blades is modelled using linear combination of the first two flap modes and the first lag mode. The Foundation is modelled as a clamped condition with all six degrees of freedom constrained.

Additionally, a Single-Rotor wind turbine with a rated power of 6MW is modelled in SIMPACK in order to compare the modal response of the Quad-Rotor wind turbine to a Single-Rotor turbine with equal swept area. The Single-Rotor wind turbine is a linearly scaled version of the NREL5MW machine. The salient properties of both the Quad-and Single-Rotor wind turbines are summarized in Table 1.

Table 1. Gross Properties of Single-and Quad-Rotor wind turbine.

Parameters	Quad-Rotor	Single-Rotor
Rating	4×1.5MW	6MW
Control	Variable Speed, Collective Pitch	Variable Speed, Collective Pitch
Rotor, Hub Diameter	70m, 3.5m	140m, 7.0m
Hub Height	(top rotors)136.75m / (bottom rotors)63.25m	100m
Rated Wind Speed	11.4m/s	11.4m/s
Rated Rotor Speed	21.8rpm	10.9rpm
Rated Tip Speed	80 m/s	80 m/s
Overhang, Shaft Tilt, Precone	3.3m, 0°, 2.5°	5.5m, 0°, 2.5°

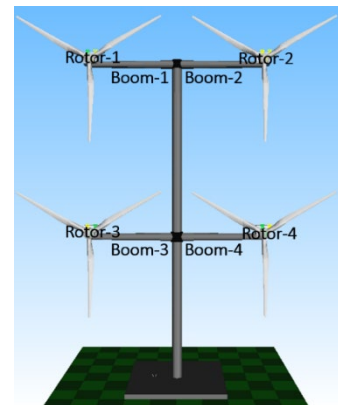
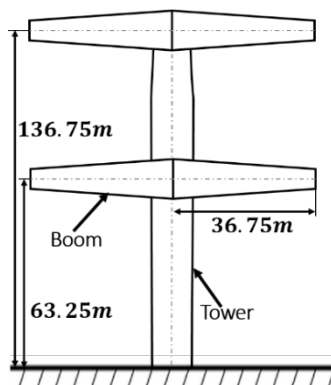


Figure 1a. Dimensions of the Tower-Boom model. Figure 1b. Quad-Rotor wind turbine model in SIMPACK.

3. System Dynamics and Multi-Blade Coordinate Transformation

The linearization of the wind turbine model is carried out at an equilibrium point for given operational parameters (e.g. mean wind speed, rotor speed) to capture the geometric nonlinearities of the system. However, the present analysis considers wind turbines rotating in vacuum. This means that the blade rotation is the only contributor to the system's nonlinear behaviour. Simpack is used to linearize the Quad-Rotor wind turbine and extract the mass (M), gyroscopic (G), and stiffness (K) matrix of the

The Eigenvector of each mode is studied to identify the mode shapes of a Quad-Rotor wind turbine. Table 2 shows the Eigenvectors of the tower modes, normalized by the largest Eigenvector per mode. Looking at Table 2, the two columns with the heading ‘Component Modes’ contain the individual mode shapes that make up the Quad-Rotor model. For example, the tower is modelled using 5 modes (two fore-aft and side-side modes plus a torsion mode) and the booms are modelled using 5 modes (two up-down and fore-aft modes plus a torsion mode). The fore-aft and up-down mode of the booms describe respectively the out-of-plane and in-plane deformation of the booms, with in-plane defined as the plane containing the tower-boom assembly. The MBC transformation allows for the blade modes to be represented in terms of the rotor modes as shown in Table 2, where the 1st flap-wise modes of the three blades are given by 1st flap-wise collective (Coll), pitch, and yaw modes; the 1st lag-wise modes of the three blades are given respectively by 1st lag-wise collective, progressive (Prog), and regressive (Reg) modes.

Table 2. Normalized Eigenvector magnitude for modes with natural frequency between 0.12-0.57Hz @ 0 (rpm).

Component Modes		1 st SS Tower	1 st FA Tower	1 st TR Tower	2 nd SS Tower	2 nd FA Tower	Component Modes		1 st SS Tower	1 st FA Tower	1 st TR Tower	2 nd SS Tower	2 nd FA Tower
5 Tower modes	1st Fore-Aft	0.03	1	0.01	0.03	1	Rotor-1	1st Flap-Coll	0	0	0.03	0	0.03
	1st Side-Side	1	0.03	0.42	1	0.03		1st Flap-Pitch	0	0	0	0.01	0.05
	2nd Diag+45	0.03	0.02	0.02	0.68	0.72		1st Flap-Yaw	0	0	0.02	0	0.01
	2nd Diag-45	0.02	0.02	0.01	0.53	0.93		1st Lag-Coll	0	0	0	0	0
	1st Torsion	0	0	1	0	0		1st Lag-Prog	0	0	0	0	0
5 modes Boom-1	1st Up-Down	0	0	0.01	0.14	0.01	1st Lag-Reg	0	0	0	0	0	
	1st Fore-Aft	0	0.01	0.15	0.01	0.11	Rotor-2	1st Flap-Coll	0	0	0.03	0	0.03
	1st Torsion	0	0	0.01	0.01	0.02		1st Flap-Pitch	0	0	0	0.01	0.05
	2nd Up-Down	0	0	0	0.01	0		1st Flap-Yaw	0	0	0.02	0	0.01
	2nd Fore-Aft	0	0	0.01	0	0		1st Lag-Coll	0	0	0	0	0
1st Up-Down	0	0	0.01	0.14	0.02	1st Lag-Prog		0	0	0	0	0	
5 modes Boom-2	1st Fore-Aft	0	0.01	0.15	0.01	0.11	1st Lag-Reg	0	0	0	0	0	
	1st Torsion	0	0	0.01	0.01	0.02	Rotor-3	1st Flap-Coll	0	0	0.02	0	0.07
	2nd Up-Down	0	0	0	0.01	0		1st Flap-Pitch	0	0	0	0.01	0.05
	2nd Fore-Aft	0	0	0.01	0	0		1st Flap-Yaw	0	0	0.01	0	0.04
	1st Up-Down	0	0	0	0.08	0		1st Lag-Coll	0	0	0	0	0
1st Fore-Aft	0	0	0.08	0	0.27	1st Lag-Prog		0	0	0	0	0	
5 modes Boom-3	1st Torsion	0	0	0.01	0.01	0.03	1st Lag-Reg	0	0	0	0	0	
	2nd Up-Down	0	0	0	0	0	Rotor-4	1st Flap-Coll	0	0	0.02	0	0.07
	2nd Fore-Aft	0	0	0	0	0.01		1st Flap-Pitch	0	0	0	0.01	0.05
	1st Up-Down	0	0	0.01	0.08	0.03		1st Flap-Yaw	0	0	0.01	0	0.04
	1st Fore-Aft	0	0	0.08	0.01	0.27		1st Lag-Coll	0	0	0	0	0
1st Torsion	0	0	0.01	0.01	0.03	1st Lag-Prog		0	0	0	0	0	
5 modes Boom-4	2nd Up-Down	0	0	0	0	0	1st Lag-Reg	0	0	0	0	0	
	2nd Fore-Aft	0	0	0	0	0.01							

Looking at Table 2, the magnitude of the Eigenvectors highlighted in red show (per mode shape) which modes of the Quad-Rotor turbine are significantly excited which helps to identify the mode shape. For example, the column with the label ‘1st SS Tower’ shows that the tower’s 1st Side-Side mode has the largest Eigenvector, indicating that the mode shape shown in the first column of Table 2 is identified as the 1st Side-Side tower motion with the remaining components moving as rigid bodies. This is further clarified using Figure 3, where the mode shapes corresponding to the Eigenvectors shown in Table 2 are presented. The order of the mode shapes in Figure 3 (from left to right) corresponds to the Eigenvectors of Table 2 (left to right).

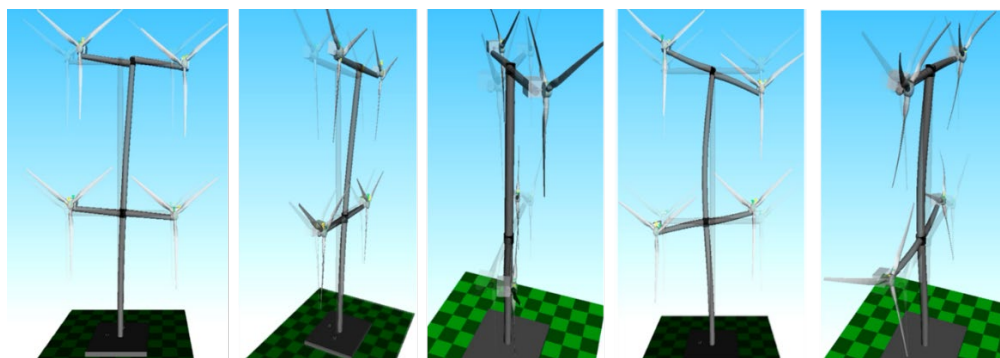


Figure 3. Mode shapes for modes with modal frequency between 0.12-0.57Hz @ 0 (rpm).

Going back to the Campbell diagram in Figure 2, the first thing to notice is that the Quad-Rotor turbine has more modes that interact with 3p excitation (before rated speed) compared to the Single-Rotor machine; due to increased number of flexible components on the Quad-Rotor turbine compared to the Single-Rotor turbine. The natural frequency of the tower’s 1st FA and SS modes reduced from 0.25Hz for the Single-Rotor tower to 0.13Hz for the Quad-Rotor tower. This is the result of combined masses of booms and nacelles that the Quad-Rotor tower carries in addition to a longer tower (compared to the Single-Rotor wind turbine) to accommodate 4 rotors arranged in a double T-format. Furthermore, the 1st FA and SS modes of the Quad-Rotor tower intersect with the 1p and 3p excitation, even if the natural frequency is kept the same as for the Single-Rotor tower to 0.25Hz; since the rated speed of the smaller rotors on the Quad-Rotor turbine is double the rated speed on the Single-Rotor turbine. Moreover, the modal frequency of the 1st torsion mode of the Quad-Rotor tower is reduced to 0.17Hz compared to 0.52Hz for the Single-Rotor tower. This is the result of placing the nacelles on the Quad-Rotor turbine farther away from the tower. This results in the Quad-Rotor structure being soft-soft to the first tower modes in that both 1p and 3p blade passing frequency interact with the tower’s natural frequency (1st FA, SS, and Torsion mode) below the rated speed. However, these modes should be sufficiently damped since the aerodynamic thrust is the primary load component that excites the tower fore-aft and torsion mode which is a highly damped force on wind turbines.

Table 3. Normalized Eigenvector magnitude for modes with natural frequency between 0.76-0.81Hz @ 0 (rpm).

Component Modes	Mode 1 TBR	Mode 2 TBR	Mode 3 TBR	Mode 4 TBR	Component Modes	Mode 1 TBR	Mode 2 TBR	Mode 3 TBR	Mode 4 TBR		
5 Tower modes	1st Fore-Aft	0.02	0.8	0.16	0.03	Rotor-1	1st Flap-Coll	0.16	0.02	0.01	0.08
	1st Side-Side	0.65	0.02	0.01	1		1st Flap-Pitch	0.22	0.83	0.54	0.46
	2nd Diag+45	0.52	0.18	0.3	0.7		1st Flap-Yaw	0.16	0.01	0.02	0.07
	2nd Diag-45	0.41	0.24	0.41	0.53		1st Lag-Coll	0.01	0	0	0
	1st Torsion	0.01	0	0	0.02		1st Lag-Prog	0.01	0.05	0.03	0.03
5 modes	1st Up-Down	0.09	1	0.78	0.25	Rotor-2	1st Lag-Reg	0.01	0.01	0.01	0.01
	1st Fore-Aft	0.51	0.09	0.01	0.24		1st Flap-Coll	0.16	0.02	0.01	0.08
	1st Torsion	0.07	0.21	0.14	0.11		1st Flap-Pitch	0.22	0.83	0.55	0.45
	2nd Up-Down	0.01	0.04	0.03	0.01		1st Flap-Yaw	0.16	0.02	0.01	0.07
Boom-1	2nd Fore-Aft	0.02	0.01	0	0.01	Rotor-3	1st Lag-Coll	0.01	0	0	0.01
	1st Up-Down	0.04	1	0.78	0.27		1st Lag-Prog	0.01	0.05	0.03	0.03
	1st Fore-Aft	0.52	0.02	0.09	0.22		1st Lag-Reg	0.01	0.01	0.01	0
Boom-2	1st Torsion	0.07	0.21	0.14	0.11	Rotor-4	1st Flap-Coll	0.31	0.05	0.06	0.14
	2nd Up-Down	0	0.04	0.03	0.01		1st Flap-Pitch	0.66	0.67	0.68	0.39
	2nd Fore-Aft	0.02	0	0.01	0.01		1st Flap-Yaw	0.29	0.04	0.08	0.18
5 modes	1st Up-Down	0.32	0.8	1	0.9	Rotor-1	1st Lag-Coll	0.02	0	0	0.01
	1st Fore-Aft	1	0.18	0.14	0.38		1st Lag-Prog	0.04	0.04	0.04	0.02
	1st Torsion	0.21	0.18	0.17	0.09		1st Lag-Reg	0.02	0.01	0.01	0
	2nd Up-Down	0.01	0.03	0.04	0.03		1st Flap-Coll	0.31	0.05	0.05	0.14
Boom-3	2nd Fore-Aft	0.04	0.01	0	0.01	Rotor-2	1st Flap-Pitch	0.65	0.67	0.67	0.4
	1st Up-Down	0.42	0.81	0.96	0.87		1st Flap-Yaw	0.3	0.06	0.06	0.17
	1st Fore-Aft	0.96	0.09	0.24	0.48		1st Lag-Coll	0.02	0	0	0.01
5 modes	1st Torsion	0.21	0.18	0.16	0.09	Rotor-3	1st Lag-Prog	0.04	0.04	0.04	0.03
	2nd Up-Down	0.02	0.03	0.04	0.03		1st Lag-Reg	0.01	0.01	0.01	0.02
	2nd Fore-Aft	0.04	0	0.01	0.02						

Table 3 shows the Eigenvector of modes in Figure 2 with modal frequency between 0.76-0.81 Hz (shown in yellow). The columns in Table 3 represent, from left to right, mode shapes with increasing modal frequency. In addition, Figure 4 portrays the mode shapes that correspond to the Eigenvectors of Table 3, with the Eigenvector of the left most column corresponding to the left most mode shape in Figure 4. Examining the Eigenvectors in Table 3 with the corresponding mode shapes shown in Figure 4, the column with the label ‘Mode 1 TBR’ represents a mode shape consisting of the 2nd side-side tower motion coupled with the 1st fore-aft motion of the top and bottom booms and flap-wise pitch mode of the bottom rotors. The column with the label ‘Mode 2 TBR’ and the corresponding mode shape in Figure 4 (second from left) represent a mode shape consisting of the 1st fore-aft tower mode coupled with the 1st (in-phase) up-down mode of the top and bottom booms together with flap-wise pitch mode of all rotors. The column with the label ‘Mode 3 TBR’ in Table 3 (with the corresponding mode shape, third from the left in Figure 4) represents a mode shape that consists of a slight 2nd fore-aft tower mode coupled with the 1st (out-of-phase) up-down mode of the booms and flap-wise pitch mode of the rotors.

The column with the label ‘Mode 4 TBR’ and the corresponding mode shape given in Figure 4 (right most) is identified as 2nd side-side tower mode coupled with 1st up-down mode of the bottom booms together with the flap-wise pitch mode of all rotors. The First three modes from Table 3 are dominated by elastic boom deformation while the last mode (Mode 4 TBR) is dominated by elastic tower deformation. It is observed, from the Campbell diagram of the Quad-Rotor turbine presented in Figure 2, that the modes with modal frequency shown in yellow decrease with increasing rotor speeds; particularly, the mode with the label ‘Mode 1 TBR’ decreases significantly with increasing rotor speed. This mode shape is sensitive to centrifugal stiffening due to presence of the flap-wise pitch mode of the bottom rotors. The remaining modes in Table 3 change slightly with increasing rotor speed.

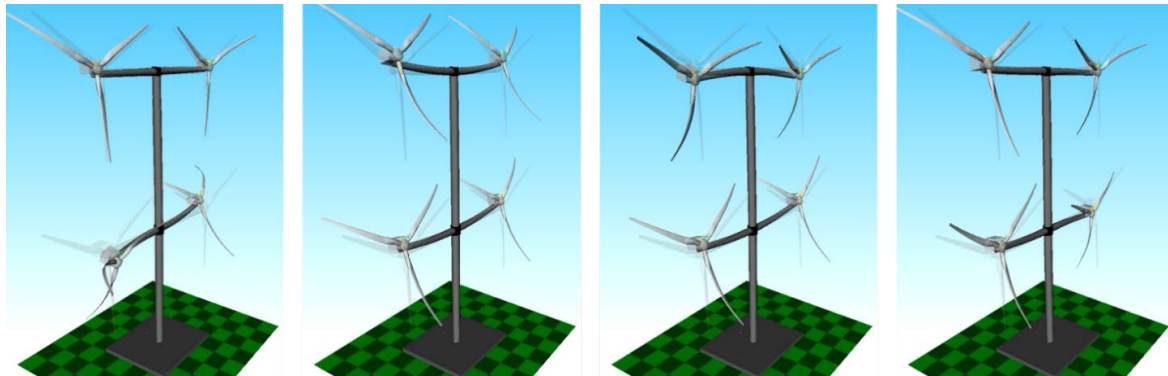


Figure 4. Mode shapes for modes with modal frequency between 0.76-0.81Hz @ 0 (rpm).

The Eigenvectors in Table 4 correspond to the modes shown in the Campbell diagram of the Quad-Rotor turbine with the modal frequency between 0.91-0.96 Hz (shown in orange in Figure 2). The Eigenvector of the mode shapes presented in Table 4 are arranged with increasing modal frequency (from left to right). The modes shapes corresponding to the Eigenvector in Table 4 are displayed in Figure 5, with increasing modal frequency from left to right. The Eigenvector with the label ‘Mode 5 TBR’ and the corresponding mode shape (1st from left) shown in Figure 5 may be classified as a mode shape composed slightly of the 2nd side-side tower mode coupled with the 1st torsion mode of the top booms and flap-wise pitch of the top rotors.

The column with the label ‘Mode 6 TBR’ in Table 4 (plus the corresponding mode shape, 2nd from left in Figure 5) is identified as 1st fore-aft tower mode coupled with a combination of the 1st (in-phase) up-down and torsion mode of all booms together with the 1st flap-wise pitch mode of all rotors. The Eigenvector with the label ‘Mode 7 TBR’ plus the corresponding mode shape in Figure 5 (3rd from left) represent a mode comprised of the 1st up-down and torsion modes of the bottom booms coupled with the 1st flap-wise pitch mode of the bottom rotors. The last mode in Table 4 with the Label ‘Mode 8 TBR’ with the corresponding mode shape in Figure 5 (4th from left) is defined as a mode comprised of the 1st (out-of-phase) up-down mode of all booms coupled with the 1st flap-wise pitch mode of all rotors.

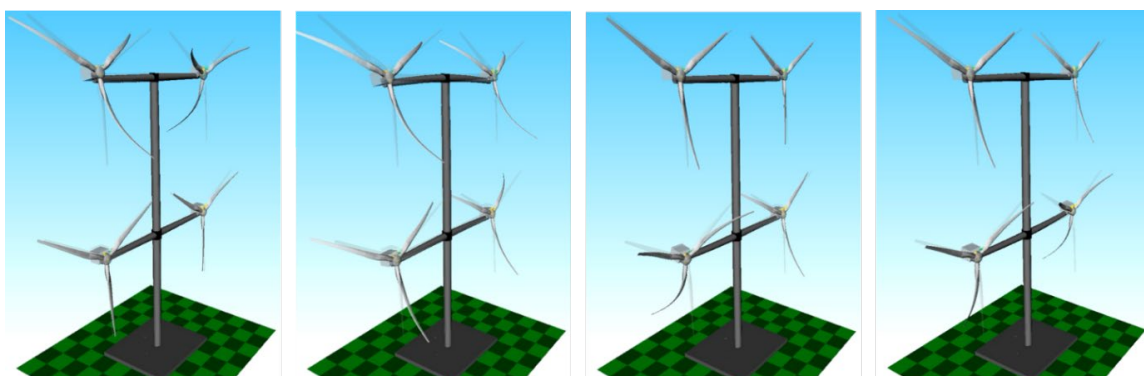


Figure 5. Mode shapes for modes with modal frequency between 0.91-0.96Hz @ 0 (rpm).

Going back to the Campbell diagram of the Quad-Rotor turbine in Figure 2, the modal frequency of the modes shown in orange (between 0.91-0.96Hz at 0 rpm) change (increase or decrease) with increasing rotor speed. The mode with the label ‘Mode 5 TBR’ decreases significantly with increasing rotor speed while Mode 8 TBR increases significantly with increasing rotor speed. The remaining two modes (Mode 6 TBR and Mode 7 TBR) decrease slightly with increasing rotor speed. Centrifugal stiffening of the rotors with increasing rotor speed play a role since the modes in Figure 5 include flap-wise pitch mode of the rotors.

Table 4. Normalized Eigenvector magnitude for modes with natural frequency between 0.91-0.96Hz @ 0 (rpm).

Component Modes	Mode 5 TBR	Mode 6 TBR	Mode 7 TBR	Mode 8 TBR	Component Modes	Mode 5 TBR	Mode 6 TBR	Mode 7 TBR	Mode 8 TBR		
5 Tower modes	1st Fore-Aft	0.01	0.91	0.01	0.04	Rotor-1	1st Flap-Coll	0.02	0.06	0.05	0
	1st Side-Side	0.33	0.03	0.19	0.01		1st Flap-Pitch	1	1	0.15	0.62
	2nd Diag+45	0.44	0.22	0.2	0.18		1st Flap-Yaw	0.08	0.09	0.07	0
	2nd Diag-45	0.34	0.29	0.16	0.22		1st Lag-Coll	0	0	0	0
	1st Torsion	0.02	0	0.01	0		1st Lag-Prog	0.06	0.06	0.01	0.04
5 modes	1st Up-Down	0.04	0.52	0.05	0.25	1st Lag-Reg	0	0	0	0	
	1st Fore-Aft	0.08	0.1	0.11	0.02	Rotor-2	1st Flap-Coll	0.02	0.06	0.05	0.01
	1st Torsion	0.16	0.14	0.02	0.09		1st Flap-Pitch	1	1	0.17	0.62
2nd Up-Down	0	0.02	0	0.01	1st Flap-Yaw		0.07	0.09	0.07	0	
2nd Fore-Aft	0	0	0	0	1st Lag-Coll		0	0	0	0	
1st Up-Down	0.03	0.51	0.03	0.25	1st Lag-Prog		0.06	0.06	0.01	0.04	
5 modes	1st Fore-Aft	0.08	0.15	0.11	0.01	1st Lag-Reg	0.01	0.01	0	0	
	1st Torsion	0.16	0.14	0.02	0.08	Rotor-3	1st Flap-Coll	0.06	0.1	0.08	0.04
	2nd Up-Down	0	0.02	0	0.01		1st Flap-Pitch	0.12	0.66	0.96	1
2nd Fore-Aft	0	0.01	0	0	1st Flap-Yaw		0.07	0.15	0.15	0.07	
1st Up-Down	0.17	0.35	0.24	0.39	1st Lag-Coll		0	0.01	0	0	
5 modes	1st Fore-Aft	0.14	0.19	0.22	0.13		1st Lag-Prog	0.01	0.04	0.06	0.06
	1st Torsion	0.02	0.1	0.13	0.13	1st Lag-Reg	0	0.01	0.01	0.01	
	2nd Up-Down	0.01	0.01	0.01	0.01	1st Flap-Coll	0.06	0.1	0.08	0.03	
5 modes	2nd Fore-Aft	0.01	0.01	0.01	0.01	Rotor-4	1st Flap-Pitch	0.12	0.66	1	0.94
	1st Up-Down	0.16	0.33	0.28	0.39		1st Flap-Yaw	0.07	0.14	0.16	0.07
	1st Fore-Aft	0.16	0.23	0.2	0.07		1st Lag-Coll	0	0.01	0	0
5 modes	1st Torsion	0.02	0.1	0.13	0.12		1st Lag-Prog	0.01	0.04	0.06	0.06
	2nd Up-Down	0	0.01	0.01	0.01		1st Lag-Reg	0.01	0.01	0.01	0
	2nd Fore-Aft	0.01	0.01	0.01	0						

5. Conclusion

This paper presents the modal analysis of a Quad-Rotor wind turbine (comprised of 4 rotors arranged in double T-format) and identifies the new modes or possible instability modes that are otherwise not present on a Single-Rotor wind turbine. Multi-Blade Coordinate transformation scheme is adapted to a Quad-Rotor wind turbine to write the system’s equation of motion in fixed-coordinate frame followed by Eigenvalue analysis to determine the natural frequency and mode shapes. The Campbell diagrams of a 6MW Quad-Rotor wind turbine and a Single-Rotor wind turbine with equal swept area is presented.

Results indicate that the Quad-Rotor turbine is soft-soft to the first tower modes (fore-aft, side-side, and torsion). This is due to the combined masses of the booms and nacelle that the tower carries and a longer tower (compared to a single rotor with equal swept area) to accommodate 4 rotors arranged in double T-format, lowering the natural frequency of the tower. Furthermore, the 1st FA and SS modes of the Quad-Rotor tower will intersect the 1p and 3p excitation, even if the natural frequency is kept the same as for the Single-Rotor tower; since the rated speed of the smaller rotors on the Quad-Rotor turbine is double the rated speed on the Single-Rotor turbine. Moreover, the low natural frequency of the tower’s torsional modes is the result of placing the nacelles farther away from the tower. However, these modes should be sufficiently damped since the aerodynamic thrust is the primary load component that excites the tower fore-aft and torsion mode which is a highly damped force on wind turbines. Finally, the modes with low natural frequency other than the tower modes are comprised of tower, boom, and blade modes. Therefore, due to the presence of blade modes, the modal frequency of these modes increases or decreases with increasing rotor speed due to the effect of centrifugal stiffening.

Future work consist of including the aerodynamic loads to the Quad-Rotor model and perform an aeroelastic stability analysis to extract the damped modal frequencies and damping ratios of the Quad-

Rotor wind turbine operating between cut-in and cut-out wind speed. Particularly, the damping ratios need to be examined to check for possible under-damped modes. For example, the damping coefficients of the soft-soft 1st tower (FA, SS, and Torsion) modes are examined to check if those modes are sufficiently damped.

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