An Analysis of Classical and Alternate Hexacopter Configurations with Single Rotor Failure

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ABSTRACT

This study examines the operation of a hexacopter in hover and forward flight conditions with a single rotor failure. A classical configuration, with adjacent rotors spinning in opposite directions, and an alternate configuration are considered. The simulation model used in this study calculates aerodynamic forces (thrust, drag and side force) and moments (pitching and rolling moments, and torque) at the rotor hub using blade element theory coupled with a finite-state dynamic inflow model to capture the rotor induced velocities. Failure of various rotors is considered individually and an understanding is developed of how the aircraft trims post-failure. For the alternate configuration hexacopter, if one of four rotors (out of six) fails, the aircraft can be trimmed in hover as well as in forward flight, and is fully controllable. But recovery from failure of one of the other two rotors for fully trimmed flight is impossible. For the classical configuration hexacopter, if any of the forward facing rotors fail, the aircraft can be trimmed in forward flight and is fully controllable. If an aft rotor fails, the aircraft cannot be trimmed, but could be turned around to orient the failed rotor forward. While the classical configuration can always be trimmed in hover by turning off the rotor diametrically opposite rotor to the failed rotor, the aircraft is not independently controllable about all axes due to a rank deficient control matrix. Thus, in the event of a rotor failure, the classical configuration hexacopter could cruise back to base and land, but not maintain a sustained hover. Power penalties of up to 23% were observed in the event of failure due to the increased induced and profile drag on the operational rotors.

INTRODUCTION

Multi-copters are becoming increasingly popular amongst hobbyists, as well as for commercial and military applications. In lieu of traditional rotor pitch controls found on a conventional helicopter, these aircraft utilize fixed pitch rotors and variable RPM control with distributed electric propulsion. As with conventional helicopters, multi-copters require at least four independent controls to operate. For a quadcopter, these controls correspond to the RPM of each of the four rotors, and in combination provide the thrust, and the pitch, roll and yaw moments required for any flight condition. With control redundancy available on multi-copters with more than four rotors (hexacopters, octocopters, etc.), several researchers have considered the implications of this redundancy on fault tolerance.

Recent work by Achtelik et al. (Ref. 1) and Du et al. (Ref. 2) shows that while hexacopters with single rotor failure in hover can be trimmed by turning off the diametrically opposite rotor, this would result in a rank-deficient control matrix, implying that an aircraft operating in such a mode cannot be controlled independently about all axes. Achtelik et al. (Ref. 1) suggest that the system would be fully controllable if the rotor diametrically opposite to the failed rotor could spin in either direction, but this conclusion is based on the diametrically opposite rotor generating only a torque without any accompanying thrust (an impossibility for fixed pitch rotors). Furthermore, their study disregards the complex aerodynamics of a rotor in reverse flow. Du et al. (Ref. 2) demonstrate a degraded control scheme and the ability to land a hexacopter with a failed rotor by relaxing the yaw equilibrium constraint.

In Ref. 3, Schneider et al. consider hexacopters in the classical configuration (Fig. 1), as well as in an alternate configuration where adjacent rotors are not all spinning in opposite directions (Fig. 2). By examining various rotor failures in hover, and the combination of forces and moments that the remaining rotors can generate, the authors conclude that the alternate configuration hexacopter would be controllable in more instances of rotor failure than the classical configuration.

Unlike Refs. 1 and 3 which focus primarily on identifying and understanding controllability in hover condition in the event of individual rotor failure(s), Falconi and Holzapfel (Ref. 4) present an adaptive controller that compensates for rotor failure without relying on explicit knowledge of the fault, provided the aircraft is controllable and flyable post-failure. Their simulations demonstrate a hexacopter with

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front rotor failure executing a level coordinated turn with the use of their controller.

Mueller and D'Andrea (Ref. 5) generate a relaxed hover solution for multi-copter aircraft that allows for the aircraft to be trimmed without being able to maintain yaw balance.

All of the studies cited above use only the simplest models for multi-copter simulations, where each rotor generates only a thrust and a torque which are assumed to vary quadratically with rotor speed. Such simple models fail to capture many aspects of rotor aeromechanics as shown by Niemiec and Gandhi (Ref. 6) and would not be applicable in forward flight conditions since they neglect, for example, the aerodynamic drag and pitching moments generated at each rotor hub. In the present study, a modified version of the higher fidelity model developed in Ref. 6 is used to analyze rotor failure on a hexacopter in both hover *and forward flight* conditions. Both the classical configuration as well as the alternate configuration hexacopter proposed in Ref. 3 are considered, and the study focuses on a physical understanding of the mechanisms of how trimmed flight is achieved after specific rotor failures, as well as why certain failures cannot be compensated for.

MODELING

The dynamic model implemented for the hexacopter uses summation of forces to determine accelerations of the aircraft, while accounting for the influence of gravity, fuselage drag (modeled as a cylinder), and the aerodynamic forces and moments at each rotor hub. The rotor blades are themselves assumed to be rigid, and the blade sectional aerodynamic lift, drag and pitching moments are determined using blade element theory, with a 3x4 (10-state) Peters-He dynamic wake model (Ref. 7) used to calculate rotor induced velocities. The sectional aerodynamic forces and moments are integrated along the blade span and around the azimuth to obtain the rotor hub forces (thrust, drag and side force), and moments (torque, pitching and rolling moments). A failed rotor is assumed to produce no forces and moments and its inflow states are ignored.

Aircraft trim involves satisfying the three force- and three moment-equilibrium equations. When four independent controls are available (as in the case of a quad-copter), these controls, along with the aircraft pitch and roll attitude, constitute the six trim variables. The system of six aircraft nonlinear equilibrium equations is solved for these six trim variables using the Newton-Raphson method. To solve the system

$$f(u) = 0 \tag{1}$$

An initial guess u_0 is assumed, and it is updated using the formula

$$u_{n+1} = u_n - J^{-1} f(u_n) \tag{2}$$

Where J is the Jacobian matrix, defined as

$$J_{ij} = \left.\frac{\partial f_i}{\partial u_j}\right|_{u=u_n} \tag{3}$$

In the case of a fully functioning hexacopter there are eight trim variables (six rotor speeds, and aircraft pitch and roll attitude). Any time there are more than six trim variables J is not square, and so J^{-1} is not defined. In this case the pseudoinverse method can be used to find a trim solution. Here the Moore-Penrose pseudoinverse (J^+) is used instead of the classical Jacobian matrix inverse in the Newton-Raphson method:

$$\begin{gathered} u_{n+1} = u_n - J^+ f(u_n) \\ J^+ = J^T (J J^T)^{-1} \end{gathered} \tag{4}$$

Note that for invertible matrices, the pseudoinverse is identical to the classical matrix inverse.

In the event of a single rotor failure on a hexacopter five independent controls remain to trim the aircraft. Since the aircraft requires four controls, there is one redundant control in this case. Instead of using the pseudoinverse method, the one-dimensional trim space can be explored by parametrically varying the speed of a single rotor (not diametrically opposed to the failed rotor). This leaves four independent controls, and thus a unique trim solution that can be found using the classical Newton-Raphson method.

TRIM RESULTS

The baseline hexacopter on which the simulations in this study are based uses 2-bladed, 10 inch diameter rotors and has a total aircraft gross weight of 2 kg. Key rotor geometry and aircraft details are given in Table 1. The hexacopter is operated in the vertex-first orientation and both classical as well as alternate configurations (Figs. 1 and 2) are analyzed.

Table 1: Aircraft Details

Parameter	Value
Rotor Radius	12.44 cm
Root Pitch	21.5°
Tip Pitch	11.1°
Root Chord	2.53 cm
Tip Chard	0.98 cm
Boom Length	30.48 cm
Motor/Rotor Mass	82 g

Fully Operational Flight

As a point of reference, the performance of the hexacopters was first analyzed with all rotors fully operational. Figures 3 and 4 present the variation of individual rotor speeds versus forward flight speed, for the classical and alternate hexacopter configurations, respectively. The solutions, obtained using the pseudoinverse method, correspond to the power optimal trim solutions presented in Ref. 8 for the classical hexacopter. For both the classical as well as the alternate configurations, all six rotors spin at identical speeds in hover. Moving to forward flight, the classical hexacopter exhibits a clear symmetry with the front rotors slowing down and the aft rotors speeding up to generate the required nose-down pitching moment (Fig. 3). The increase in speed of aft rotors 3 and 5 is the same as the reduction in speed of front rotors 2 and 6 (about the mean rotor speed), while the increase in RPM of vertex rotor 4 and the reduction for vertex rotor 1 (about the mean) is twice that of the changes for rotors 2, 3, 5 and 6. These results are consistent with Ref. 8. On the other hand, for the alternate configuration hexacopter, there is no clear pattern or symmetry to the individual rotor speeds in forward flight (Fig. 4). Generally, aft rotors (3, 4 and 5) spin faster than forward rotors (1, 2 and 6) to produce a nose down pitching moment. But there is neither any symmetry with respect to the longitudinal axis (rotors 2 and 6 do not operate at the same RPM, and neither do rotors 3 and 5, as was the case with the classical configuration), nor are the changes in vertex rotor speeds (1 and 4) twice those of the inner rotors (2, 3, 5 and 6) about the mean rotor speed. Overall, the controls for the classical configuration hexacopter in forward flight are far simpler, and more intuitive, than those of the alternate configuration.

In Ref. 8, Niemiec and Gandhi defined orthogonal multi-rotor control modes (producing pure thrust, pitch, roll or yaw) for classical hexacopters. Similar orthogonal modes can be derived for the alternate configuration hexacopter, as well. As an example, Figs. 5 and 6 show the roll control mode for the classical and alternate configuration hexacopters, respectively. To roll left, the classical configuration simply slows down the two left rotors (2 and 3) while speeding up the two right rotors (5 and 6). The orthogonal roll control mode (producing no net thrust, pitch or yaw moment) for the alternate configuration, on the other hand, is highly complex and non-intuitive, requiring various rotors to slow down and speed up to various degrees (Fig. 6). The larger changes in individual rotor RPM required (relative to the conventional hexacopter) limit the maximum roll moment that can be generated on the alternate configuration. Calculations show that the maximum roll left moment generated on the alternate configuration hexacopter in hover is about 33% lower than possible for the classical configuration due to rotor 5 reaching its maximum speed limit.

The power requirement for both the classical and alternate configurations, versus airspeed, is shown in Fig. 7. Clearly, with all rotors operational, the two configurations show no difference in power requirement, the complex control modes for the alternate configuration notwithstanding. The power consumptions at hover and 5 m/s are marked on the figure for later comparisons with rotor failure cases.

Rotor Failure in Hover, Classical Configuration

For the classical configuration hexacopter (Fig. 1), assuming rotor 1 to have failed and varying the speed of rotor 2 parametrically, the speeds of the remaining rotors (3, 4, 5, and 6) can be calculated uniquely. The individual rotor speeds (versus speed of rotor 2) are presented in Fig. 8, for the hover condition. From Fig. 8 it is evident that the trim solution requires a diametric rotor pairing. Thus rotor 4 (diametrically opposite to the failed rotor 1) turns off, rotors 2 and 5 operate at the same speed, and the same is true for rotors 3 and 6. Also shown on Fig. 8 is power requirement versus the speed of rotor 2. The minimum power solution corresponds to the case where all operating rotors (2, 3, 5 and 6) are spinning at an identical speed of 6500 RPM (as indicated by the dashed vertical line on Fig. 3). The minimum power requirement of 172 watts is 22% greater than the fully operational hexacopter and is attributed to increases in both induced power (higher disk loading with only four operational rotors generating the required thrust) as well as profile power (the four operational rotors are spinning at 6500 RPM compared to a speed of 5325 RPM when all rotors are operational as seen in Fig. 3).

Although the aircraft trims by turning off the rotor diametrically opposite the failed rotor and operating the other four rotors at identical speeds, this configuration differs from a quad-copter in that adjacent (operational) rotors do not spin in opposite directions. Specifically, both the front rotors (2 and 6) are clockwise-spinning and both the rear rotors (3 and 5) spin counter-clockwise. If the aircraft was required to generate a nose-down pitching moment (to counteract a gust, for example), it would speed up the rear rotors and slow down the front rotors, but in doing so would generate a net yaw moment. Thus for rotor 1 (or rotor 4) failure, generation of a pitching moment is inextricably coupled to the generation of a yaw moment. Unavailability of independent four-axis control corroborates with the rank-deficiency in the control sensitivity matrix previously discussed in the literature (Ref. 1). This attribute of the classical hexacopter being trimmable, but not controllable, in hover holds even if the rotors are not operating at the minimum power state shown in Fig. 8. Furthermore, due to axisymmetric considerations, the ability to trim but not provide independent four-axis control in hover applies to a classical hexacopter regardless of which rotor fails.

Rotor Failure in Hover, Alternate Configuration

Next, rotor 1 failure is considered for the alternate configuration hexacopter (Fig. 2) in hover condition. Once again, the speed of rotor 2 is parametrically varied and the speeds of the remaining four rotors are calculated uniquely. Figure 9 shows the variation in the individual rotor speeds versus rotor 2 speed, and the dashed vertical line again corresponds to the minimum power solution. Unlike the classical hexacopter, rotor 4 (diametrically opposite failed rotor 1) is not turned off. Additionally, there is no evidence of diametric rotor pairing (diametrically opposite rotors do not spin at the same speeds). At minimum power operation, the front rotors (2 and 6), which are adjacent to the failed rotor, spin faster than the aft rotors (3, 4 and 5) to maintain pitch balance. Furthermore, the counter-clockwise rotor-6 spins faster than clockwise rotor-2 to help maintain yaw balance since there are only two functional counter-clockwise spinning rotors after the failure of rotor 1. Of the aft rotors, the single clockwise spinning rotor-4 spins faster than the two counter-clockwise spinning rotors (3 and 5). Rotor 3 spins faster than 5 to counteract the rolling moment generated by rotor 6 spinning faster than rotor 2. The alternate configuration hexacopter with rotor 1 failure is fully controllable in hover (with no rank deficiency in the control sensitivity matrix seen in the classical configuration). The minimum power solution on Fig. 9 requires 165 watts, which is 17% higher than the fully operational case. This power penalty is lower than the case of single rotor failure on the classical configuration since five rotors are operational instead of four.

Figure 10 presents trim results in hover when rotor 2 fails on the alternate configuration hexacopter. This time the speed of rotor 1 is parametrically varied and the speeds of the remaining four rotors are calculated uniquely. The results in Fig. 10 are essentially identical to those in Fig. 9. At minimum power operation, the rotors 1 and 3 adjacent to the failed rotor 2 spin faster than the rotors (4, 5 and 6) on the opposite side (of an axis dividing rotors 1, 2 and 3 from rotors 4, 5 and 6). Furthermore, the clockwise rotor-3 spins faster than counter-clockwise rotor-2 to help maintain yaw balance (since there are only two clockwise rotors after the failure of rotor 2). Of the rotors 4, 5, and 6, rotor-5 spins fastest since it is one of only two clockwise spinning rotors. Rotor 6 spins faster than 4 to counteract rotor 3 spinning faster than rotor 1. From the discussion above it is clear that for the alternate configuration hexacopter the mechanism by which trim is achieved is the same in the case of rotor 1 or rotor 2 failures in hover, and this results in identical power requirement, as well. The control sensitivity matrix is also full-rank in the case of rotor 2 failure.

For the alternate configuration hexacopter, rotor 6 failure in hover is analyzed next using a similar approach of parametrically varying the speed of one of the other rotors (rotor 3) and uniquely calculating the speed of the remaining four rotors. The trim control results are shown in Fig. 11. The minimum power solution (dashed vertical line) once again has the two rotors (1 and 5) adjacent to the failed rotor 6 spinning fastest, with rotor 1 spinning faster than rotor 5 since there are only two counter-clockwise rotors operational. This is similar to the results seen in Figs. 9 and 10 for failed rotors 1 and 2. Failed rotor 6 differs from rotor 1 and 2 failures in that the rotors 2, 3 and 4, across from the failed rotor and its adjacent rotors (1, 6 and 5) do not have alternating spin directions (as was the case for failure of rotors 1 and 2). Of these three rotors (2, 3 and 4), rotor 4 now spins fastest since it is one of only two operational rotors spinning counter-clockwise, and it balances rotor 1. Rotor 2 follows and rotor 3 (diametrically opposed to the failed rotor) spins slowest. Recall that for failure of rotors 1 or 2, the diametrically opposite rotor spun faster than the rotors adjacent to it, but for failure of rotor 6 the reverse is true, (with the diametrically opposite rotor 3 spinning slower than the rotors 2 and 4 adjacent to it). The minimum power corresponding to rotor 6 failure is 162 watts, 15% higher than the fully operational case, and the control sensitivity matrix again exhibits no rank deficiency (so the aircraft is independently controllable about all axes). It should be noted that for failed rotor 6, in the limit rotor 3 can be slowed down all the way to zero speed. This would reduce the aircraft to operating like a classical quad-copter with adjacent operational rotors (1, 2, 4 and 5) having opposite spin directions. However, the power requirement to operate in quad-copter mode would be larger.

In hover, failure of rotor 3 on the alternate configuration hexacopter results in a functionally identical situation to that seen in the case of failed rotor 6.

The failure of rotors 4 or 5 on the alternate configuration hexacopter does not allow the aircraft to trim in hover. Consider, for example, the failure of rotor 4. The remaining two counter-clockwise rotors (1 and 6) at the front of the aircraft would try to speed up to achieve yaw-balance, but this would lead to a nose-up pitching moment and a roll-left moment. Speeding up rotors 2, 3 and 5 to counteract the pitch and roll moments would again lead to yaw imbalance. Thus, it is impossible to simultaneously satisfy equilibrium about all axes in the case of rotor 4 failure, and the same holds true for rotor 5. Although there is one numerical solution to trimming the alternate configuration hexacopter in hover in the event of rotor 4 or rotor 5 failure, this is *not considered to be a physically viable option*. This solution requires turning off all rotors except rotors 3 and 6. While the aircraft would be in pitch, roll, and yaw balance, the two rotors would have to spin extremely fast (beyond the maximum motor torque capability) to carry the aircraft weight and the aircraft would additionally be uncontrollable. It is interesting to note that on the alternate configuration hexacopter when rotors 4 or 5 fail, the two adjacent rotors to the failed rotor spin in the same direction. In contrast, when rotors 1, 2, 3 or 6 fail, the two adjacent rotors spun in opposite directions.

Rotor Failure in Forward Flight, Classical Configuration

In forward flight, the hexacopter assumes a nose down pitch attitude so that the thrust from the rotors can provide a propulsive force to overcome aerodynamic drag. Rotor failure results are presented at a moderate flight speed of 5 m/s.

Figure 12 shows trim results for rotor 1 failure on the classical configuration hexacopter. Unlike the hover condition, rotor-4 (opposite to the failed rotor) is seen to operate at a non-zero speed. The diametric pairing of opposite rotors seen in hover is observed in forward flight as well, and at the minimum power condition (dashed vertical line) rotors 2, 3, 5 and 6 operate at mostly similar speeds. In forward flight each counter-clockwise spinning rotor experiences a roll-left hub moment and each clockwise spinning rotor experiences a roll-right hub moment (Ref. 6) due to higher dynamic pressure on the advancing blades. The hub roll moments of rotors 2, 3, 5 and 6 mostly cancel, and the smaller roll-right moment from the slower rotor 4 is cancelled by the slightly larger thrust produced by the right rotors (5 and 6) than the left rotors (2 and 3). In forward flight each rotor also produces a nose-up aerodynamic pitching moment at the hub (Ref. 6), and as a consequence of the nose down pitch attitude, the gravity vector additionally produces a nose up pitching moment (about a reference point in the rotor plane right above the aircraft CG). The sum of these nose-up moments is predominantly counteracted by the nose-down pitching moment generated by the thrust of rotor 4. The thrusts of rotors 2, 3, 5 and 6 do not produce a significant net pitching moment. While the hub torques for rotors 2, 3, 5 and 6 mostly cancel, in forward flight the clockwise spinning forward rotors 2 and 6 produce a rightward side-force, while counter-clockwise spinning rear rotors produce a leftward side-force, looking down on the aircraft (Ref. 6). The side forces from rotors 2, 3, 5 and 6 collectively produce a yaw-right moment on the aircraft which counteracts the yaw-left hub torque at rotor 4 (and the yaw-left moment from rotor 4 side-force). The minimum-power operating state after rotor 1 failure requires 153 watts, about 20% larger than the baseline (128 watts, shown on Fig. 7).

In forward flight, with rotor 2 failed, Fig. 13 shows the rotational speeds of the other rotors (calculated by varying the RPM of rotor 1 parametrically). As in the case of rotor 1 failure, the opposite rotor (5) to the failed rotor (2) operates at a non-zero speed, albeit significantly slower than the other operational rotors (3, 4, 6 and 1). Of the other rotors, aft rotors 3 and 4 spin faster than rotors 6 and 1. Together with rotor 5, thrusts from rotors 3 and 4 contribute to the nose down pitching moment required in forward flight. Rotor 3 operating faster than rotor 6 also helps with roll balance. The hub yaw moments from rotors 3 and 4 cancel as do the hub yaw moments from rotors 1 and 6. The nose-left yaw moment from the side forces on rotors 1 and 4 exceed the nose right yaw moment from the side forces on rotors 3 and 6 and serve to balance out the nose-right hub torque and the nose-right yaw moments from rotor 5 drag and side-force. The minimum-power operating state after rotor 2 failure requires 158 watts, about 23% larger than the fully-operational case.

In forward flight, rotor 6 failure is similar to rotor 2 failure and the aircraft can be similarly trimmed. With five operating rotors (unlike hover, where the rotor opposite to the failed rotor was turned off to trim the aircraft), there is no rank deficiency and the classical configuration hexacopter offers independent control about all axes. Failure of aft rotors 3, 4, and 5 does not yield a trim solution for the aircraft in forward flight. This is primarily due to the inability to generate the requisite nose-down pitching moment in forward flight with an aft rotor failure, while simultaneously meeting roll and yaw equilibrium. Consider, for example, the case of rotor 4 failure in forward flight. Yaw moment balance would require the two remaining clockwise rotors (2 and 6) to speed up, but this would generate a nose-up pitching moment. Speeding up the aft rotors to generate the required nose-down moment would lead to imbalance in yaw moment. In the event of aft rotor failure in forward flight the best strategy would be to turn the aircraft around so the failed rotor is oriented forward.

Rotor Failure in Forward Flight, Alternate Configuration

Figures 14, 15 and 16 show the speeds of the operational rotors for failure of rotors 1, 2, and 6, respectively, on the alternate configuration hexacopter at 5 m/s forward flight speed. It is perhaps most instructive to compare these figures to rotor controls for the same failures in hover condition.

For rotor 1 failure, comparing Fig. 14 to the hover controls in Fig. 9 reveals that the controls are mostly similar with the most evident change being the increase in speed of rotor 4 over the entire range to generate the nose-down pitching moment required in forward flight. For rotor 2 failure, a comparison of Fig. 15 to the hover controls presented in Fig. 10 similarly suggests that the key difference is the aft rotor 4 spinning faster to generate a nose-down pitching moment. A similar observation can made for rotor 6 failure, by comparing controls at 5 m/s (Fig. 16) to the hover controls (Fig. 11). In all three front rotor failure cases, minimum power required at 5 m/s forward speed (corresponding to the vertical dashed lines in Figs. 14-16) is about 150-151 watts, an increase of up to 17-18% from the fully operational case.

When the aft rotor 3 on the alternate configuration hexacopter fails in forward flight, the controls do not change in the same manner. Figure 17 shows the controls at 5 m/s. While these controls seem qualitatively similar to the controls in hover (shown in Fig. 18), the key difference appears to be that the forward rotor 1 slows down to induce a net nose-down pitching moment (rather than the aft rotor 4 speeding up, as was the case with any front rotor failures). The power requirement of 153 watts is only slightly higher than that required for failures of rotors 1, 2, or 6.

In the case of failure of rotors 1, 2, 3 and 6 in forward flight on the alternate configuration hexacopter, the aircraft remains fully controllable, as it did in hover.

If rotors 4 and 5 fail, as was the case in hover, no viable trim solution is realizable in forward flight. It should be noted that rotating the aircraft to have rotors 4 or 5 in any other position (for example, toward the front of the aircraft) does not alleviate the inability to trim the alternate configuration hexacopter.

CONCLUSIONS

Using a simulation model, this paper focuses on operation of a hexacopter in hover and forward flight conditions after single rotor failure. Both a classical configuration hexacopter with adjacent rotors spinning in opposite directions as well as an alternate configuration is considered. The simulation model calculates rotor forces and moments using blade-element theory coupled with a finite-state dynamic inflow model to represent the rotor induced velocities. As a result, the simulations accurately capture the rotor thrust, drag, pitching moment, rolling moment, side force and torque, and their variations with flight condition. For the fully operational hexacopter, the trim controls in forward flight are highly intuitive for the classical configuration, but are far more complex and unintuitive for the alternate configuration.

For the classical configuration hexacopter, when any rotor fails in hover, trim is achieved by turning off the diametrically opposite rotor, and having the four remaining rotors run at identical speeds (for minimum power operation). The resulting configuration differs from a classical quad-copter in that adjacent rotors do not spin in opposite directions. This system has a rank-deficient control sensitivity matrix in hover, implying that independent control about all axes cannot be achieved. So while a classical hexacopter with a rotor failure can be trimmed in hover, it is not fully controllable, rendering it unable, for example, to hold position when subject to disturbance.

In forward flight, the classical configuration hexacopter can be trimmed when one of the forward rotors fails. Unlike hover, the diametrically opposite rotor is no longer turned off, but operates at a low speed (relative to the other four rotors) and plays a key role in generating the required nose-down pitching moments. Importantly, in these conditions the control sensitivity matrix is of full rank, implying the aircraft is fully controllable. In the event of a rear rotor failure, it is impossible to trim the classical configuration hexacopter in forward flight. In large part, this is due to the inability to generate the required nose-down pitching moments while still satisfying equilibrium about other axes. However, the aircraft could be turned around to orient the failed rear rotor forward and cruise back to base, with the aircraft being fully controllable. A failed rotor results in a 22% power penalty in hover and a 20-23% power penalty in forward flight, relative to the fully operational cases.

Unlike the classical hexacopter where rotors alternate in spin direction, on the alternate configuration hexacopter four of the rotors have the two rotors adjacent to them spinning in opposite direction, while two rotors have the two rotors adjacent rotors spinning in the same direction. If either of these two rotors fail, the aircraft cannot be trimmed in hover or in forward flight. Thus, it would be impossible to recover from the failure of one of these two rotors. If any of the other four rotors fail the alternate configuration hexacopter can be trimmed in both hover and in forward flight, and the control sensitivity matrix is of full row rank in both cases, as well. This results in a 15-17% power penalty in hover and a 20-23% power penalty in forward flight, relative to the fully operational alternate configuration hexacopter.

In summary, for the alternate configuration hexacopter, there is a 66% probability of recovery from rotor failure, because if one of four rotors (out of six) fails, the aircraft can be trimmed in hover as well as forward flight and is fully controllable. For the classical configuration hexacopter, if any of the forward facing rotors fail, the aircraft can be trimmed in forward flight and is fully controllable. If an aft rotor fails, the aircraft cannot be trimmed, but could be turned around so the failed rotor is oriented forward. The classical configuration can be trimmed in hover, but with a rank deficient control sensitivity matrix, is not independently controllable about all axes. So in the event of a rotor failure the classical configuration hexacopter can cruise back to base and land, but not maintain a sustained hover.

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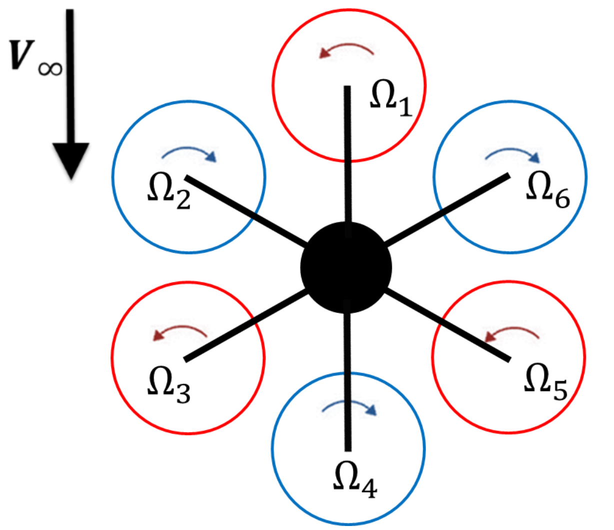

Figure 1: Classical Configuration Hexacopter

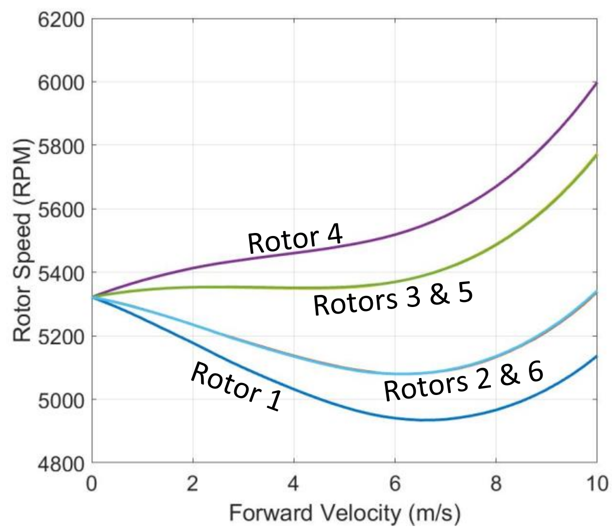

Figure 3: Fully Operational Trim Solutions for the Classical Configuration Hexacopter

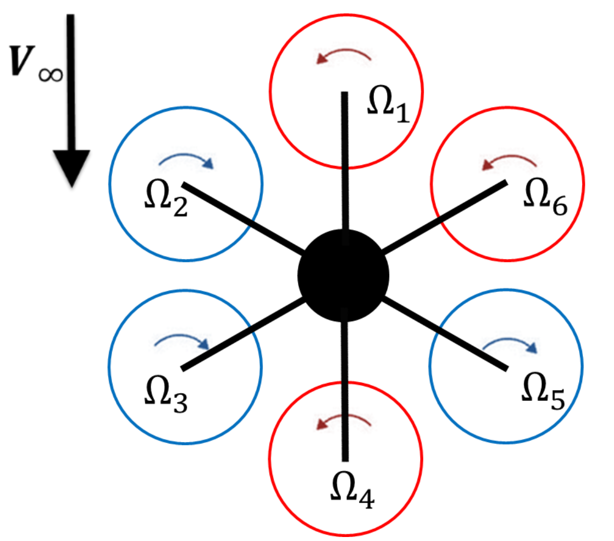

Figure 2: Alternate Configuration Hexacopter

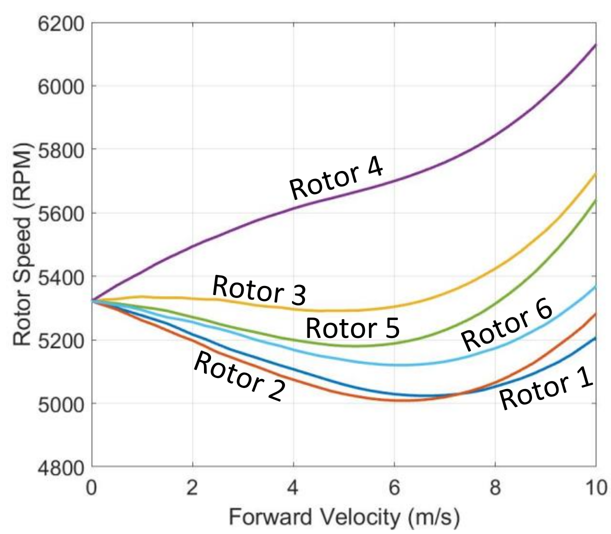

Figure 4: Fully Operational Trim Solutions for the Alternate Configuration Hexacopter

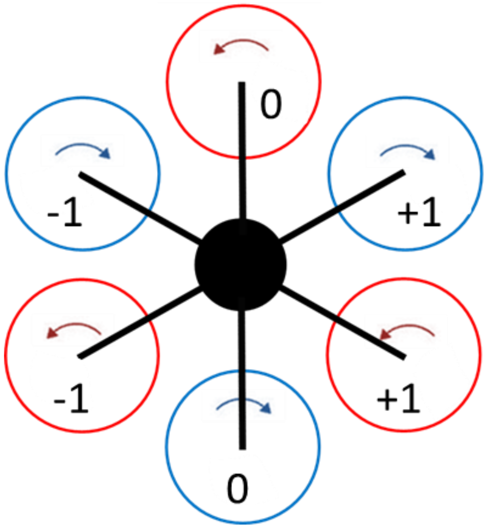

Figure 5: Roll Left Control Mode for Classical Configuration Hexacopter

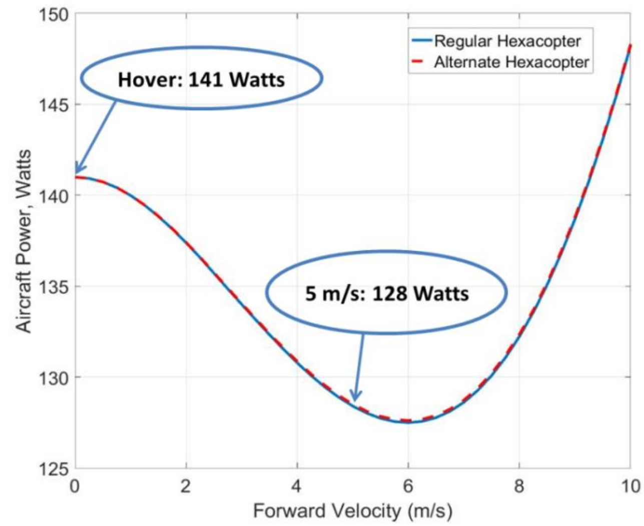

Figure 7: Power Requirement for Both Configurations versus Forward Flight Speed

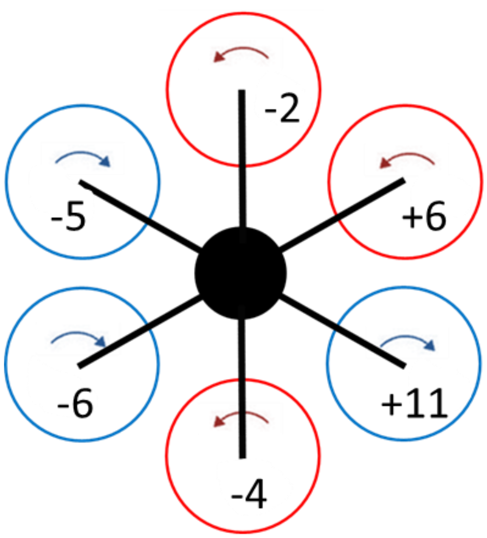

Figure 6: Roll Left Control Mode for Alternate Configuration Hexacopter

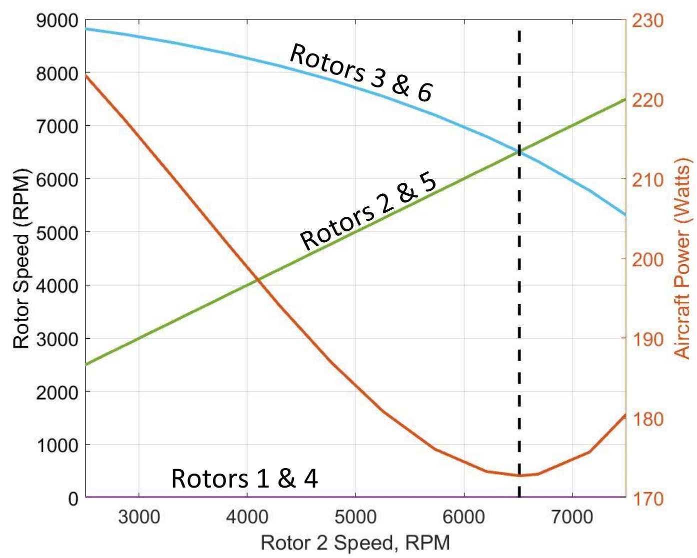

Figure 8: Rotor 1 Failure Trim Solutions in Hover for Classical Configuration Hexacopter, with Power Requirement in Red

.

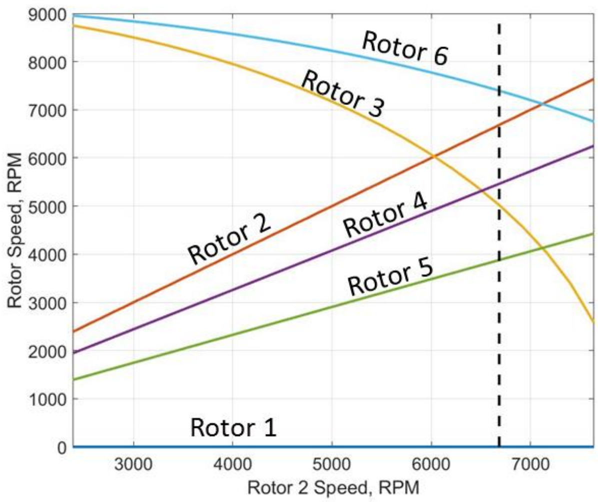

Figure 9: Rotor 1 Failure Trim Solutions in Hover for Alternate Configuration Hexacopter

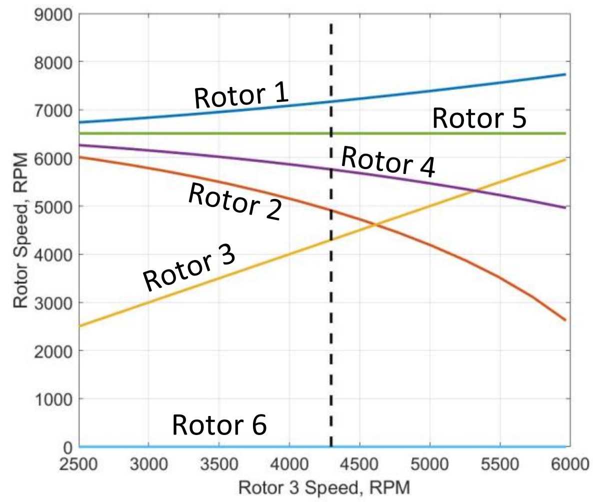

Figure 11: Rotor 6 Failure Trim Solutions in Hover for Alternate Configuration Hexacopter

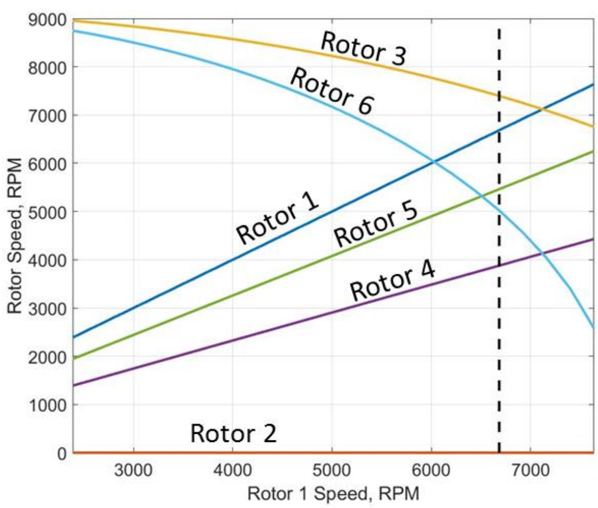

Figure 10: Rotor 2 Failure Trim Solutions in Hover for Alternate Configuration Hexacopter

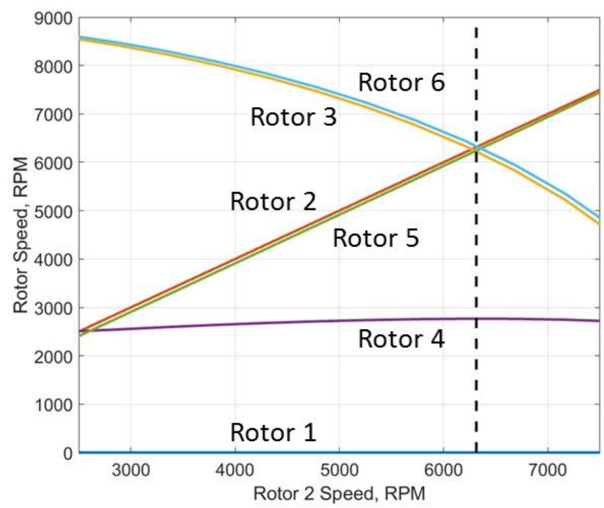

Figure 12: Rotor 1 Failure Trim Solutions at 5 m/s for Classical Configuration Hexacopter

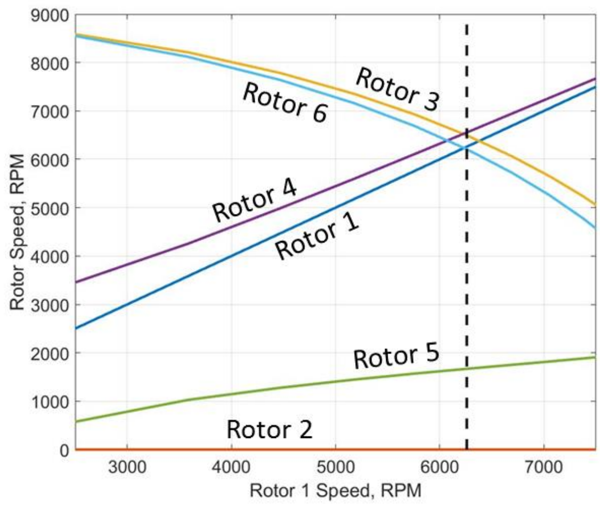

Figure 13: Rotor 2 Failure Trim Solutions at 5 m/s for Classical Configuration Hexacopter

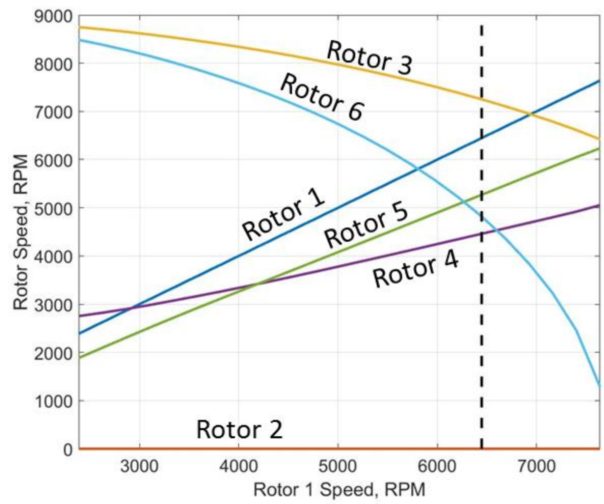

Figure 15: Rotor 2 Failure Trim Solutions at 5 m/s for Alternate Configuration Hexacopter

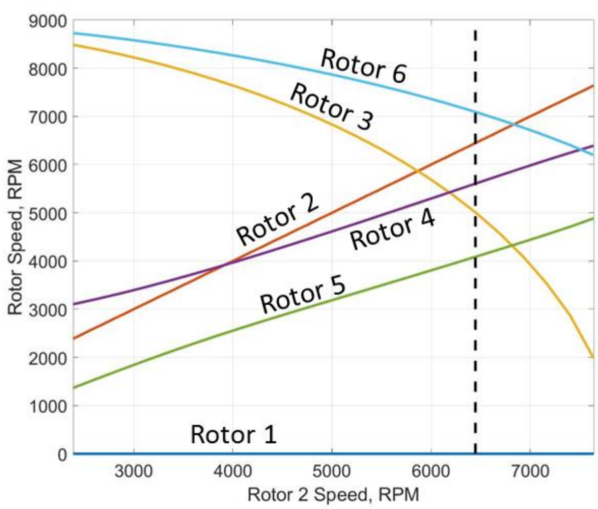

Figure 14: Rotor 1 Failure Trim Solutions at 5 m/s for Alternate Configuration Hexacopter

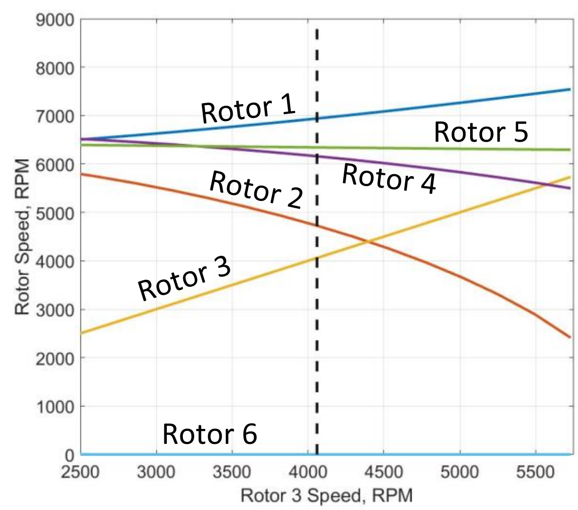

Figure 16: Rotor 6 Failure Trim Solutions at 5 m/s for Alternate Configuration Hexacopter

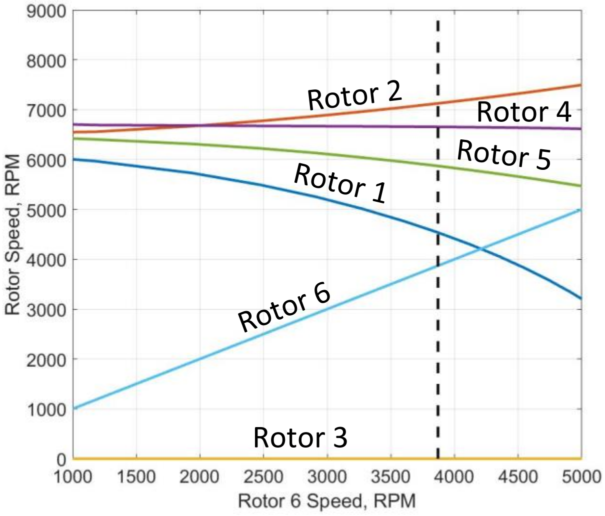

Figure 17: Rotor 3 Failure Trim Solutions at 5 m/s for Alternate Configuration Hexacopter

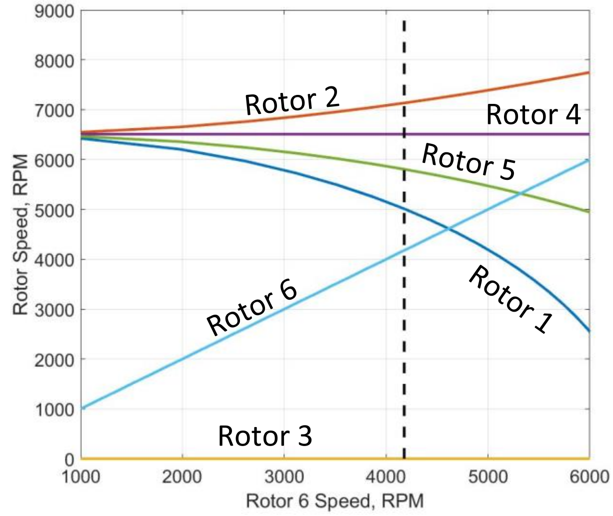

Figure 18: Rotor 3 Failure Trim Solutions in Hover for Alternate Configuration Hexacopter