

Control Reconfiguration for a Hexacopter Experiencing Single Rotor Failure

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Abstract

This study examines the operation of a classical hexacopter (with adjacent rotors spinning in opposite directions) in hover and forward flight after a single rotor failure. The simulation model used calculates aerodynamic forces (thrust, drag and side force) and moments (pitching and rolling moments, and torque) at the rotor hub using blade element theory coupled with a finite-state dynamic inflow model to capture the rotor induced velocities. Failure of various rotors is considered individually and an understanding is developed of how the aircraft trims post-failure in both vertex-first and edge-first orientations. The results show that the classical hexacopter trims in hover by turning off the rotor diametrically opposite the failed rotor, and equally speeding up the remaining four rotors for power-optimal operation. The resulting power penalty is 22% relative to the fully operational hexacopter. In hover, the control sensitivity matrix, however, is rank-deficient, implying that independent control about different axes is not available (affecting its ability to hold position or compensate for disturbances). In cruise flight, the classical hexacopter can be trimmed if any of the forward facing rotors fail. The rotor diametrically opposite to the failed rotor is slowed down but, unlike hover, does not turn off. The power penalty, varying between 20-27%, increases as the position of the failed rotor moves from forward-most to a lateral location on the aircraft. Importantly, the control sensitivity matrix in cruise flight is full-rank, implying the aircraft is fully controllable in cruise. If a rotor aft of the pitch axis fails the hexacopter cannot be trimmed in cruise, owing to the inability to provide nose-down pitching moments while simultaneously satisfying equilibrium about the other axes. Thus, in the event of a forward rotor failure, the classical hexacopter could cruise back to base, momentarily hover into a landing, but not maintain a sustained hover. For an aft-rotor failure, the best solution would appear to be turning the aircraft around so that the failed rotor faces forward.

1. Introduction

Multicopters are becoming increasingly popular with hobbyists, as well as in commercial and military applications. Enabled by distributed electric propulsion, these aircraft utilize variable-RPM (fixed pitch) rotors, and eschew the traditional rotor pitch controls found on larger conventional helicopters. Each rotor's speed is independently controlled in order to generate the required thrust and moments needed to control the aircraft. As with conventional helicopters, multicopters require a minimum of four independent controls to operate. For a quadcopter, these controls are the rotational speeds of each of its four rotors. A multicopter with more than four rotors (hexacopter, octocopter, decacopter, etc.) naturally possesses control redundancies, which provide tolerance to rotor failure.

Several groups have investigated the implications of this fault tolerance on multicopters. Recent work by Achtelik et al. (Ref. 1) and Du et al. (Ref. 2) shows that while hexacopters with single rotor failure in hover can be trimmed by turning off the diametrically opposite rotor, this would result in a rank-deficient control sensitivity matrix, implying that an aircraft operating in such a mode cannot be controlled independently about all axes. Achtelik et al. (Ref. 1) suggest that the system would be fully controllable if the rotor diametrically opposite to the failed rotor could spin in either direction, but this conclusion is based on the diametrically opposite rotor generating only a torque without any accompanying thrust (an impossibility for fixed pitch rotors). Furthermore, their study disregards the complex aerodynamics of a rotor in fully reversed flow. Du et al. (Ref. 2) demonstrate a degraded control scheme and the ability to land a hexacopter with a failed rotor by relaxing the yaw equilibrium constraint.

Fault tolerance of other configurations of hexacopters have also been considered in the literature, Schneider et al (Ref. 3) consider a classical hexacopter with adjacent rotors spinning in opposite directions as well as an alternate configuration where adjacent rotors are not all spinning in opposite directions. By examining various rotor failures in hover, and the combination of forces and moments that the remaining rotors can generate, the authors conclude that

the alternate configuration hexacopter would be controllable in more instances of rotor failure than the classical configuration.

Unlike Refs. 1 and 3 which focus primarily on identifying and understanding controllability in hover conditions in the event of individual rotor failure(s), Falconi and Holzapfel (Ref. 4) present an adaptive controller that compensates for rotor failure without relying on explicit knowledge of the fault, provided the aircraft is controllable and flyable post-failure. Their simulations demonstrate a hexacopter with front rotor failure executing a level coordinated turn with the use of their controller. Mueller and D'Andrea (Ref. 5) generate a relaxed hover solution for multicopters that allows for the aircraft to be trimmed without being able to maintain yaw balance.

All of the studies cited above use only the simplest models for multicopter simulations, where each rotor generates only a thrust and a torque which are assumed to vary quadratically with rotor speed. Such simple models fail to capture many aspects of rotor aeromechanics as shown by Niemiec and Gandhi (Ref. 6) and would not be applicable in forward flight conditions since they neglect, for example, the aerodynamic drag and pitching moments generated at each rotor hub. In the present study, a modified version of the higher fidelity model developed in Ref. 6 is used to analyze rotor failure on a classical hexacopter in both hover and forward flight conditions. Both vertex-first as well as edge-first orientations (Figs. 1 and 2) are considered, and the study focuses on a physical understanding of the mechanisms of how trimmed flight is achieved after specific rotor failures.

2. Modeling

The dynamic model implemented for the hexacopter uses summation of forces to determine accelerations of the aircraft, while accounting for the influence of gravity, fuselage drag (modeled as a cylinder), and the aerodynamic forces and moments at each rotor hub. The rotor blades are themselves assumed to be rigid, and the blade sectional aerodynamic lift, drag and pitching moments are determined using blade element theory, with a 3x4 (10-state) Peters-He dynamic wake model (Ref. 7) used to calculate rotor induced velocities. The sectional aerodynamic forces and moments are integrated along the blade span and around the azimuth to obtain the rotor hub forces (thrust, drag and side force), and moments (torque, pitching and rolling moments). A failed rotor is assumed to produce no forces and moments and its inflow states are ignored.

Aircraft trim involves satisfying the three force- and three moment-equilibrium equations. When four independent controls are available (as in the case of a quadcopter), these controls, along with the aircraft pitch and roll attitude, constitute the six trim variables. The system of six aircraft nonlinear equilibrium equations is solved for these six trim variables using the Newton-Raphson method. To solve the system:

$$f(u) = 0 \quad (1)$$

An initial guess u_0 is assumed, and it is updated using the formula

$$u_{n+1} = u_n - J^{-1}f(u_n) \quad (2)$$

Where J is the Jacobian matrix, defined as

$$J_{ij} = \left. \frac{\partial f_i}{\partial u_j} \right|_{u=u_n} \quad (3)$$

In the case of a fully functioning hexacopter there are eight trim variables (six rotor speeds, and aircraft pitch and roll attitude). Any time there are more than six trim variables J is not square, and so J^{-1} is not defined. In this case the pseudoinverse method can be used to find a trim solution. Here the Moore-Penrose pseudoinverse (J^+) is used instead of the classical Jacobian matrix inverse in the Newton-Raphson method:

$$\begin{aligned} u_{n+1} &= u_n - J^+ f(u_n) \\ J^+ &= J^T (J J^T)^{-1} \end{aligned} \quad (4)$$

Note that for invertible matrices, the pseudoinverse is identical to the classical matrix inverse.

In the event of a single rotor failure on a hexacopter, five independent controls remain to trim the aircraft. Since the aircraft requires four controls, there is one redundant control in this case. Instead of using the pseudoinverse method, the one-dimensional trim space can be explored by parametrically varying the speed of a single rotor (not diametrically opposed to the failed rotor). This leaves four independent controls, and thus a unique trim solution that can be found using the Newton-Raphson method.

3. Trim Results

Table 1: Aircraft Geometry

The baseline hexacopter on which the simulations in this study are based uses 2-bladed, 10 inch diameter rotors and has a total aircraft gross weight of 2 kg. Key rotor geometry and aircraft details are given in Table 1. The hexacopter is analyzed operating in both the vertex-first as well as the edge-first orientations (Figs. 1 and 2).

Parameter	Value
Rotor Radius	12.44 cm
Root Pitch	21.5°
Tip Pitch	11.1°
Root Chord	2.53 cm
Tip Chord	0.98 cm
Boom Length	30.48 cm
Motor/Rotor Mass	82 g

3.1 Fully Operational Flight

A fully operational classical hexacopter is first considered in order to provide a baseline for comparison after rotor failure. Figure 3 presents the variation of individual rotor speeds versus forward flight speed, for the hexacopter in the vertex-first orientation. The solutions, obtained using the pseudoinverse method, correspond to the power-optimal trim solutions presented in Ref. 8. From Fig. 3, all six rotors spin at identical speeds in hover. Moving to forward flight, a clear symmetry is observed with the front rotors slowing down and the aft rotors speeding up to generate the required nose-down pitching moment to provide propulsive thrust. The increase in speed of aft rotors 3 and 5 is the same as the reduction in speed of front rotors 2 and 6 (about the mean rotor speed), while the increase in RPM of vertex rotor 4 and the reduction for vertex rotor 1 (about the mean) is twice that of the changes for rotors 2, 3, 5 and 6. Figure 4 presents similar results for the hexacopter in the edge-first orientation. In forward flight, a slow-down of the front rotors 1 and 2 and an equivalent speed-up of the aft rotors 4 and 5 (about the mean rotor speed) is observed, for the provision of a nose-down pitching moment. The rotors 3 and 6 are also operating at identical speed (the mean rotor speed). Unlike the vertex-first case where rotors are operating at four distinct speeds in forward flight, rotors operate at only three distinct speeds in the edge-first case. The power requirement, as a function of airspeed, is identical for both the vertex-first and edge-first cases, and is shown in Fig. 5.

3.2 Rotor Failure in Hover

For the vertex-first orientation (Fig. 1), assuming rotor 1 to have failed and varying the speed of rotor 2 parametrically, the speeds of the remaining rotors (3, 4, 5, and 6) can be calculated uniquely. The individual rotor speeds (versus speed of rotor 2) are presented in Fig. 6, for the hover condition. From Fig. 6 it is evident that the trim solution requires a diametric rotor pairing. Thus rotor 4 (diametrically opposite to the failed rotor 1) turns off, rotors 2 and 5 operate at identical speeds, and the same is true for rotors 3 and 6. Also shown on Fig. 6 is power requirement versus the speed of rotor 2. The minimum power solution (indicated by the dashed vertical line on Fig. 6) corresponds to the case where all operating rotors (2, 3, 5 and 6) are spinning at an identical speed of 6500 RPM. The minimum power requirement of 172 watts is 22% greater than the fully operational hexacopter and is attributed to increases in both induced power (higher disk loading with only four operational rotors generating the required thrust) as well as profile power (the four operational rotors are spinning at 6500 RPM compared to a speed of 5325 RPM in hover when all rotors are operational as seen in Figs. 3 and 4).

Although the aircraft trims by turning off the rotor diametrically opposite the failed rotor and operating the other four rotors at identical speeds, this configuration differs from a quadcopter in that adjacent (operational) rotors do not spin in opposite directions. Specifically, both the front rotors (2 and 6) are clockwise-spinning and both the rear rotors (3 and 5) spin counter-clockwise. If the aircraft was required to generate a nose-down pitching moment (to counteract a

gust, for example), it would speed up the rear rotors and slow down the front rotors, but in doing so would generate a net yaw moment. Thus for rotor 1 (or rotor 4) failure, generation of a pitching moment is inextricably coupled to the generation of a yaw moment. In general, rotor failure in hover for the hexacopter will result in a coupling of aircraft moments dependent on the location of the failed rotor. Unavailability of independent four-axis control corroborates with the rank-deficiency in the control sensitivity matrix previously discussed in the literature (Ref. 1). This attribute of the hexacopter, of being trimmable but not fully controllable in hover, holds even if the rotors are not operating at the minimum power state shown in Fig. 6. Furthermore, due to considerations of axisymmetry, the ability to trim but not provide independent four-axis control in hover applies to a classical hexacopter regardless of which rotor fails, or aircraft orientation.

3.2 Rotor Failure in Cruise

In forward flight, the hexacopter assumes a nose down pitch attitude so that the thrust from the rotors can provide a propulsive force to overcome aerodynamic drag. Rotor failure results are presented at a moderate flight speed of 5 m/s for operation in both vertex-first and edge-first orientations.

3.2.1 Forward Rotor Failure, Vertex-First Orientation

Figure 7 shows trim results for rotor 1 failure for the hexacopter in the vertex-first orientation. Unlike the hover condition, rotor 4 (opposite to the failed rotor) is seen to operate at a non-zero speed. The diametric pairing of opposite rotors seen in hover is observed in forward flight as well, and at the minimum power condition (dashed vertical line) rotors 2, 3, 5 and 6 operate at mostly similar speeds. In forward flight each counter-clockwise spinning rotor experiences a roll-left hub moment and each clockwise spinning rotor experiences a roll-right hub moment (Ref. 6) due to higher dynamic pressure on the advancing blades. The hub roll moments of rotors 2, 3, 5 and 6 mostly cancel, and the smaller roll-right moment from the slower rotor 4 is cancelled by the slightly larger thrust produced by the right rotors (5 and 6) than the left rotors (2 and 3). In forward flight, each rotor also produces a nose-up aerodynamic pitching moment at the hub (Ref. 6), and as a consequence of the nose down pitch attitude, the gravity vector additionally produces a nose-up pitching moment (about a reference point in the rotor plane right above the aircraft CG). The sum of these nose-up moments is predominantly counteracted by the nose-down pitching moment generated by the thrust of rotor 4. The thrusts of rotors 2, 3, 5 and 6 do not produce a significant net pitching moment. While the hub torques for rotors 2, 3, 5 and 6 mostly cancel, in forward flight the clockwise spinning forward rotors 2 and 6 produce a rightward side-force, while counter-clockwise spinning rear rotors produce a leftward side-force, looking down on the aircraft (Ref. 6). As seen in Fig. 8 the side forces from rotors 2, 3, 5 and 6 collectively produce a nose-right yaw moment on the aircraft which counteracts the yaw-left hub torque from rotor 4 (and the yaw-left moment from rotor 4 side-force). The minimum-power operating state after rotor 1 failure requires 153 watts, about 20% larger than the baseline (128 watts, shown on Fig. 5).

For the failed rotor 2 case in vertex-first orientation, Fig. 9 shows the rotational speeds of the other rotors (calculated by varying the RPM of rotor 1 parametrically). As in the case of rotor 1 failure, the opposite rotor (5) to the failed rotor (2) operates at a non-zero speed, albeit significantly slower than the other operational rotors (3, 4, 6 and 1). Of the other rotors, aft rotors 3 and 4 spin faster than their diametrically opposed rotors 6 and 1. Together with rotor 5, thrusts from rotors 3 and 4 contribute to the nose down pitching moment required in forward flight. Rotor 3 operating faster than rotor 6 also assists with roll balance. The hub yaw moments from rotors 3 and 4 cancel as do the hub yaw moments from rotors 1 and 6. As indicated on Fig. 10, the nose-left yaw moment from the side forces on rotors 1 and 4 exceed the nose right yaw moment from the side forces on rotors 3 and 6 and serve to balance out the nose-right hub torque and the nose-right yaw moments from rotor 5 drag and side-force. The minimum-power operating state after rotor 2 failure requires 158 watts, about 23% larger than the fully-operational case. In cruise, rotor 6 failure is similar to rotor 2 failure and the aircraft can be similarly trimmed.

3.2.2 Forward Rotor Failure, Edge-First Orientation

Figure 11 shows trim results for rotor 2 failure for the hexacopter in the edge-first orientation. With rotor 2 turned off, the opposite rotor, 5, is seen to slow down considerably. For power-optimal operation rotors 1, 3, 4, and 6 are observed to be operating at roughly similar speeds, but rotors 3 and 4 on the left side of the aircraft do spin slightly faster than the rotors to the right to ensure roll moment equilibrium. In addition to the thrust from slower spinning

rotor 5 (diametrically opposed to the broken rotor 2), the higher thrust from aft rotor 4 spinning slightly faster than its diametrically opposed forward rotor 1 contributes to a nose-down pitching moment to put the aircraft in a nose-down attitude required in cruise (thrust from rotors 3 and 6 have no contribution to pitching moment due to their geometric positions). The sum of the hub torques from the three operational counter-clockwise rotors (1, 3, and 5) and the two clockwise rotors (4 and 6) produces a net nose-right yaw moment on the aircraft, which is further augmented by the drag and the side force of the unpaired slower-spinning rotor 5. As indicated on Fig. 12, this is cancelled by the nose-left yaw moment generated by the side-forces from rotors 1 and 4 (with the side forces from rotors 3 and 6 exerting no yaw moment on the aircraft due to their positions contribution). The minimum-power operating state after rotor 2 failure in edge-first orientation requires 155 watts, about 21% larger than the baseline. Rotor 1 failure in the edge-first orientation shows analogous behavior to that discussed above.

Figure 13 shows trim results for rotor 3 failure in the edge-first orientation. With rotor 3 turned off, the opposite rotor, 6, is seen to slow down considerably. For power-optimal operation rotor 6 slows down all the way to 735 RPM and rotors 1, 2, 4, and 5 operate between 6000-6800 RPM. As diametrically opposed rotor pairs go, aft rotor 4 spins faster than forward rotor 1, and aft rotor 5 similarly spins faster than its counterpart – forward rotor 2. The higher thrust from the faster-spinning aft rotors generates the nose-down pitching moment to put the aircraft in the required nose-down pitch attitude. The hub torques from the three clockwise-spinning rotors (2, 4, and 6) and the two counter-clockwise-spinning rotors (1 and 5) results in a net nose-left yaw moment on the aircraft. This is negated by the side forces from rotors 1, 2, 4, and 5, as indicated on Fig. 14. At the front of the aircraft, with rotor 2 spinning faster and thereby producing a larger side force than rotor 1, there is a net rightward pointing force that contributes to a nose-right yaw moment. Similarly, at the rear of the aircraft, with rotor 5 spinning faster and thereby producing a larger side force than rotor 4, there is a net leftward-pointing force that also contributes to a nose-right yaw moment. The drag from the slow-spinning rotor 6 further adds to the nose-right moment required to equilibrate the aircraft in yaw. The total rolling moment contributions from diametrically opposite rotor pairs are shown in Fig. 15. The faster clockwise rotor 4 produces a larger roll-right hub moment than the roll-left hub moment from rotor 1 – its slower counter-clockwise spinning counterpart. The larger thrust from rotor 4, relative to rotor 1, further adds to the roll-right moment. Similarly, with rotor 5 spinning faster than the diametrically opposed rotor 2, contributions from both hub roll moments, as well as from the thrust of the two rotors, result in a net roll-left moment on the aircraft. From Fig. 15 it is seen that the larger roll-right moment from the 1-4 pair than the roll-left moment from the 2-5 pair is compensated by a net roll-left moment from rotor 6 (with the roll-left moment about the aircraft center of gravity from rotor 6 thrust dominating the roll-right hub moment generated due to clockwise rotation). The minimum-power operating state after rotor 6 failure requires 162 watts, about 27% larger than the baseline. Rotor 6 failure in the edge-first orientation shows analogous behavior to that discussed above.

3.2.3 Forward Rotor Failure, Summary

With five operating rotors in cruise (unlike hover, where the rotor opposite to the failed rotor was turned off to trim the aircraft), there is no rank deficiency in the control sensitivity matrix and the classical hexacopter, operating in either vertex-first or edge-first orientation, offers independent control about all axes. Trimmed flight can be achieved with any forward rotor failure. At 5 m/s cruise speed, Fig. 16 summarizes the minimum power penalty corresponding to specific rotor failures. The power penalty is seen to progressively increase as the location of the failed rotor moves from forward-most to a more lateral position. Failure of a rotor at 0 deg (forward-most rotor in vertex-first orientation) operates with the lowest power penalty (20%), followed by a 21% power penalty for failure of a rotor at 30 deg (one of the two forward rotors in edge-first orientation). Failure of a rotor at 60 deg (rotors 2 or 6 in vertex-first orientation, see Fig. 1) incurs an even larger power penalty (23%), while failure of a rotor at 90 deg (rotors 3 or 6 in edge-first orientation, see Fig. 2) incurs the greatest power penalty (27%) among all cases considered. It should be noted that the power penalties discussed above correspond to the power-optimal solution for each rotor failure case. On Figs. 7, 9, 11 and 13, it is evident that the power penalty could be considerably greater for operations away from the optimal case.

3.2.4 Aft Rotor Failure

Failure of any aft rotor (rotors 3, 4, and 5 in vertex-first orientation, Fig. 1, or rotors 4 and 5 in edge-first orientation, Fig. 2) does not yield a trim solution for the aircraft in forward flight. This is primarily due to the inability to generate the requisite nose-down pitching moment in forward flight with an aft rotor failure, while simultaneously

meeting roll and yaw equilibrium. Consider, for example, the case of rotor 4 failure in the vertex-first orientation. Yaw moment balance would require the two remaining clockwise rotors (2 and 6) to speed up, but this would generate a nose-up pitching moment. Speeding up the aft rotors to generate the required nose-down moment would lead to imbalance in yaw moment. In the event of aft rotor failure in forward flight the best strategy would be to turn the aircraft around so the failed rotor is oriented forward. To avoid such a maneuver, the edge-first orientation may be preferable since failure of four rotors (1, 2, 3 and 6 on Fig. 2) can be tolerated in forward flight, resulting in a trimmable and fully controllable aircraft in forward flight. In contrast, the vertex-first orientation can tolerate the failure of only three rotors (1, 2 and 6 on Fig. 1) for trim and full controllability (without requiring turning of the aircraft to orient the failed rotor forward).

4. Conclusions

Using a simulation model, this paper focuses on operation of a classical hexacopter in hover and forward flight conditions after single rotor failure. Both vertex-first and edge-first orientations are considered for a 2 kg gross-weight classical hexacopter. The simulation model calculates rotor forces and moments using blade element theory coupled with a finite-state dynamic inflow model to represent the rotor induced velocities. As a result, the simulations accurately capture the rotor thrust, drag, pitching moment, rolling moment, side force and torque, and their variations with flight condition. In the study, trim controls and power requirements for the fully operational aircraft are calculated first and used as a point of reference for comparison with the failed rotor cases. For the case of a failed rotor on the hexacopter, the RPM of a second rotor is varied parametrically and a unique set of solutions is obtained for the remaining rotors. The analysis predominantly focuses on the minimum power solutions, post-failure.

In hover, when any rotor fails on the hexacopter, trim is achieved by turning off the diametrically opposite rotor, and having the four remaining rotors run at identical speeds (for minimum power operation). With the four rotors operating at a higher speed relative to the fully functional hexacopter, and carrying the weight of the entire aircraft, a resulting power penalty of 22% was observed. The hexacopter with a failed rotor and its diametrically opposite rotor turned off in hover differs from a classical quadcopter in that adjacent rotors do not spin in opposite directions. The control sensitivity matrix is rank-deficient implying that independent control about all axes cannot be achieved in hover. So while a classical hexacopter with a rotor failure can be trimmed in hover, it is not fully controllable, rendering it unable, for example, to hold position when subject to disturbance.

In cruise flight, the classical configuration hexacopter can be trimmed when one of the forward rotors fails. Unlike hover, the diametrically opposite rotor is no longer turned off, although it operates at a significantly lower speed (relative to the other four rotors). Importantly, the control sensitivity matrix in forward flight is of full rank, implying the aircraft is fully controllable. At 5 m/s cruise speed the power penalty associated with forward rotor failure varies between 20-27% relative to the fully operational case, increasing as the location of failed rotor moves from the forward-most position to lateral positions on the aircraft. In the event of a rear rotor failure, it is impossible to trim the classical configuration hexacopter in cruise flight. In large part, this is due to the inability to generate the required nose-down pitching moments while still satisfying equilibrium about other axes. However, the aircraft could be turned around to orient the failed rear rotor forward and cruise back to base, with the aircraft being fully controllable.

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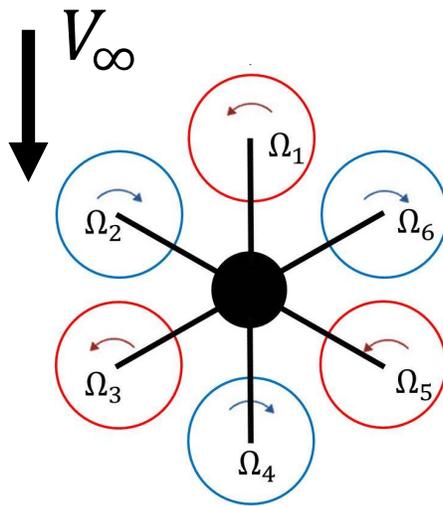


Figure 1: Classical hexacopter in vertex-first orientation

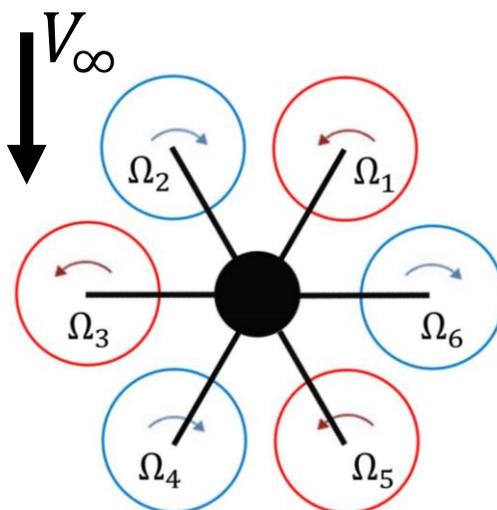


Figure 2: Classical hexacopter in edge-first orientation

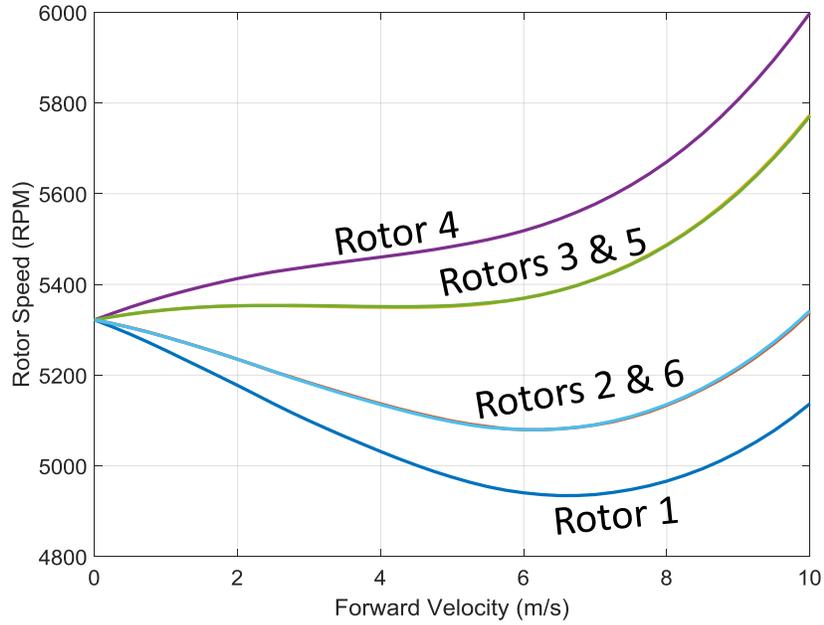


Figure 3: Trimmed Rotor Speeds vs Flight Speed for Fully Operational Hexacopter in Vertex-First Flight

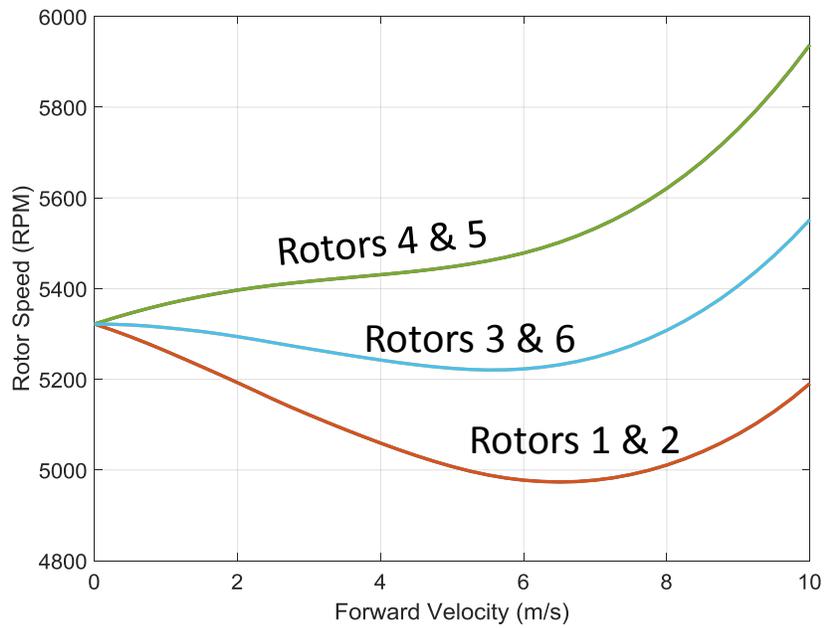


Figure 4: Trimmed Rotor Speeds vs Forward Flight Speed for Fully Operational Hexacopter in Edge-First Flight

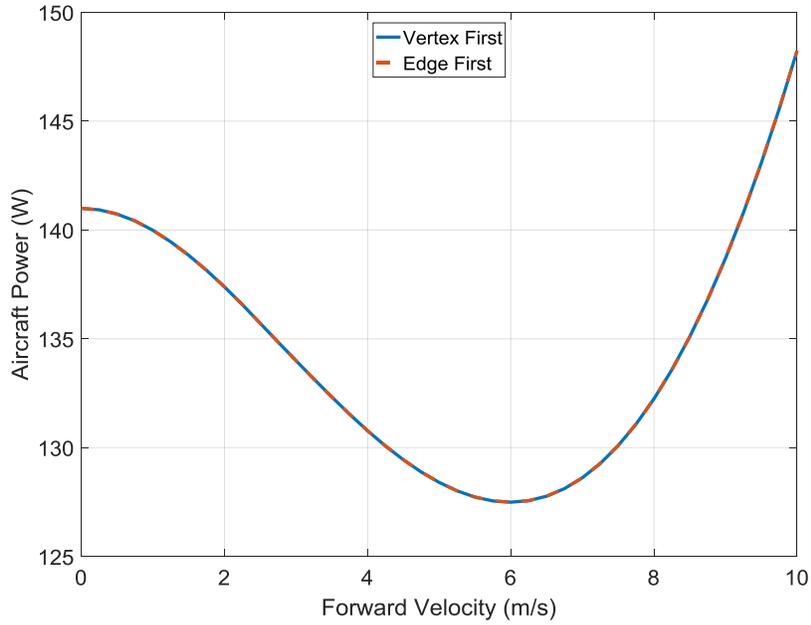


Figure 5: Aircraft Power vs Flight Speed for both Hexacopter Configurations

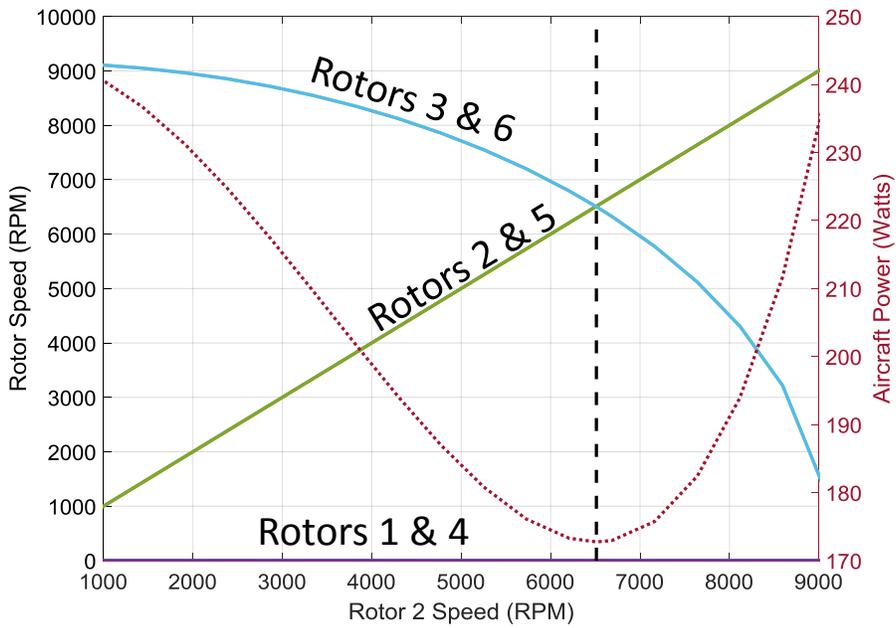


Figure 6: Trimmed Rotor Speeds for Rotor 1 Failure in Hover - Vertex First

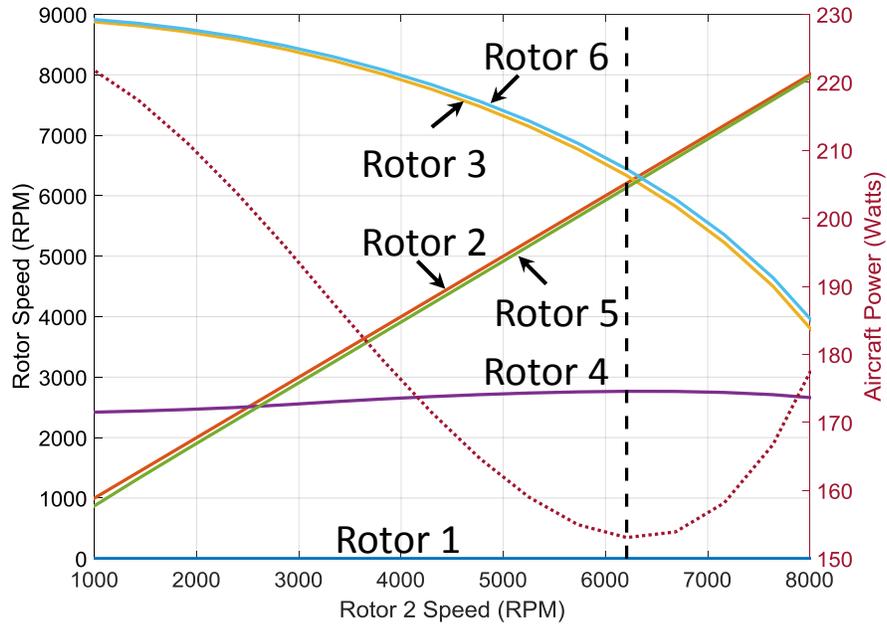


Figure 7: Trimmed Rotor Speeds for Rotor 1 Failure in Forward Flight - Vertex First

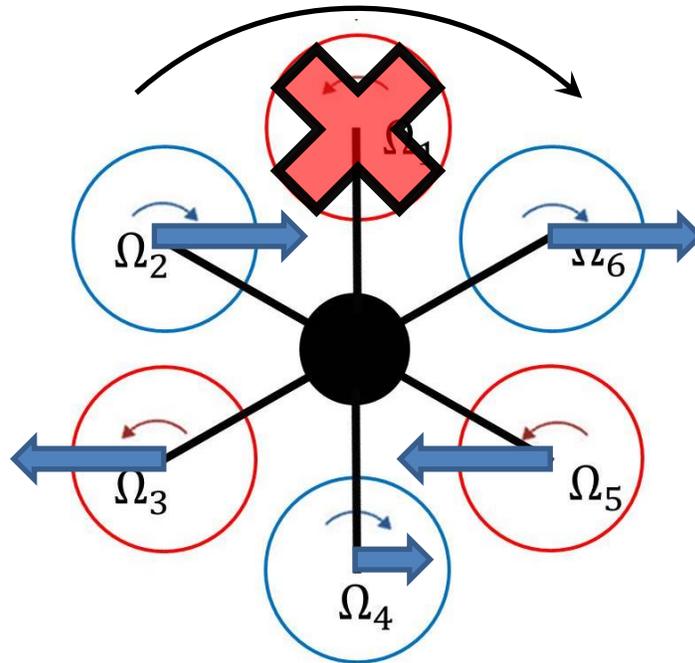


Figure 8: Rotor Side Forces, Rotor 1 Failure in Forward Flight

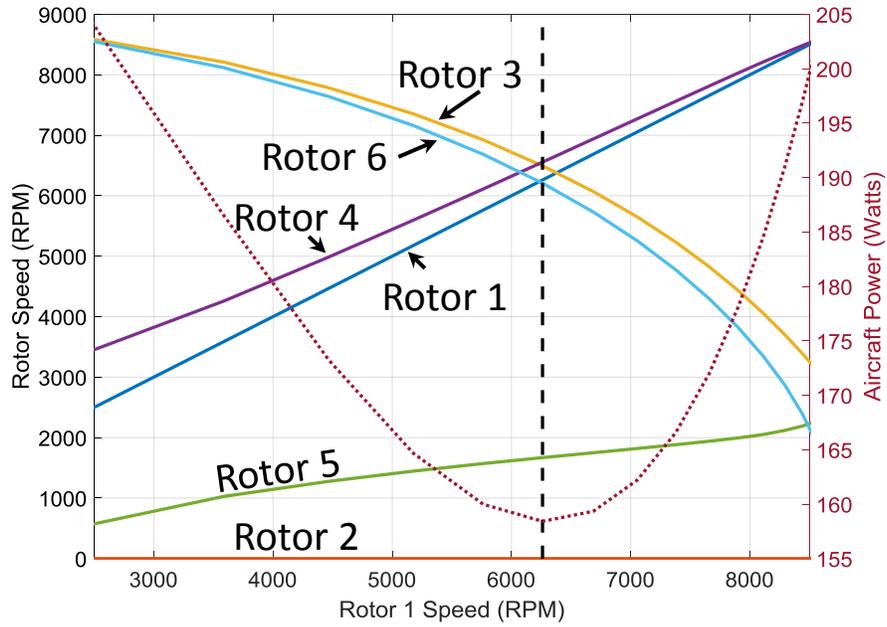


Figure 9: Trimmed Rotor Speeds for Rotor 2 Failure in Forward Flight - Vertex First

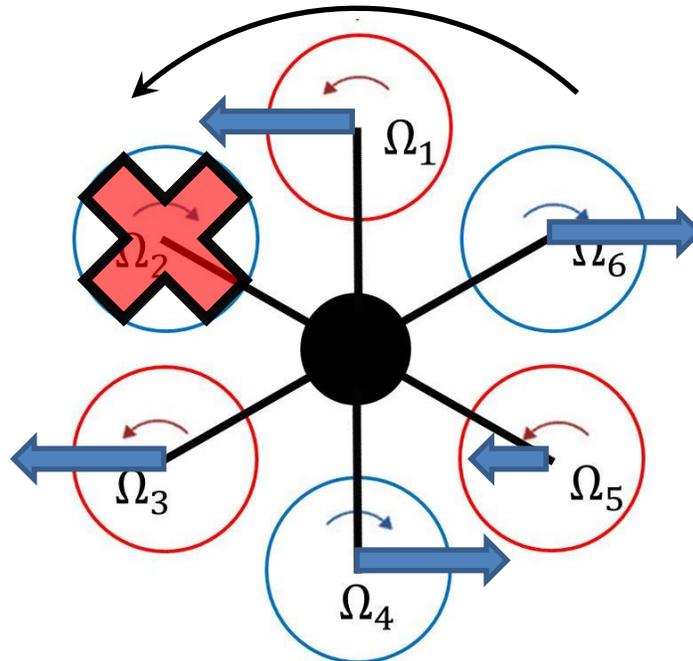


Figure 10: Rotor Side Forces, Rotor 2 Failure in Forward Flight

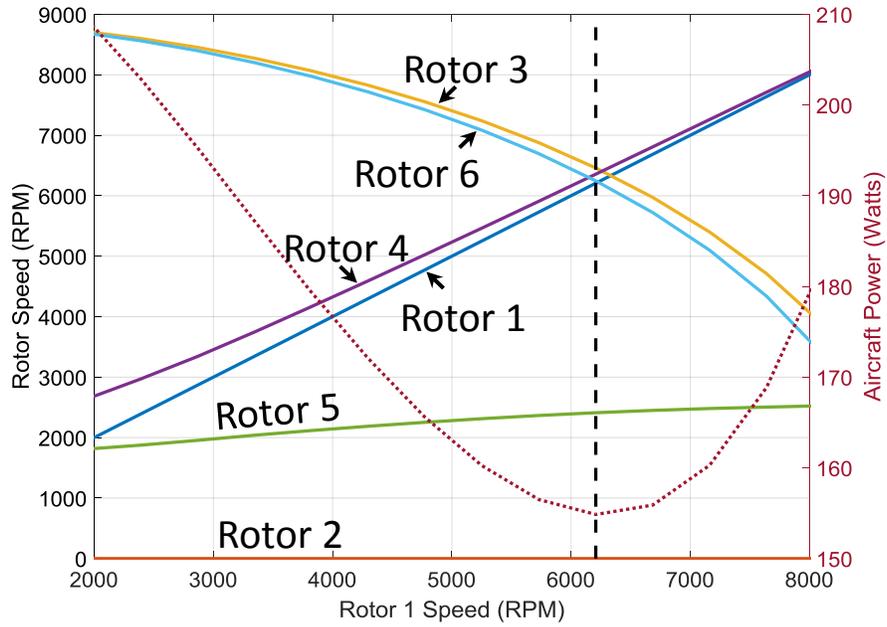


Figure 11: Trimmed Rotor Speeds for Rotor 2 Failure in Forward Flight - Edge First

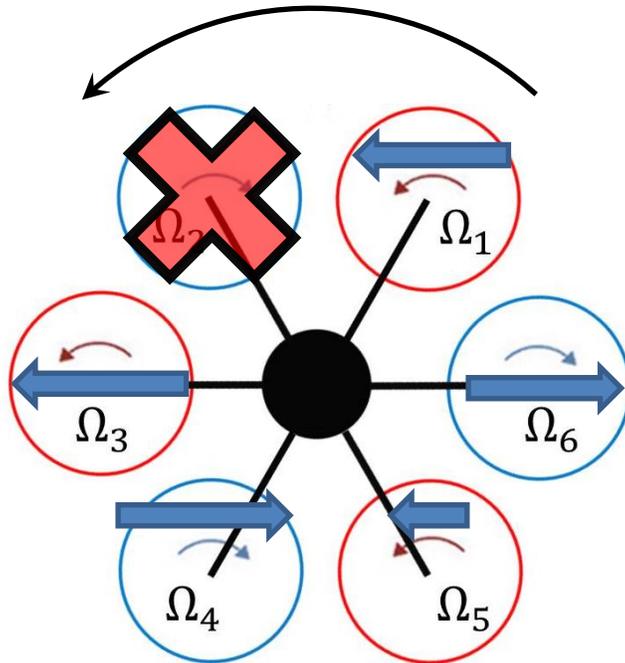


Figure 12: Rotor Side Forces, Rotor 2 Failure in Forward Flight

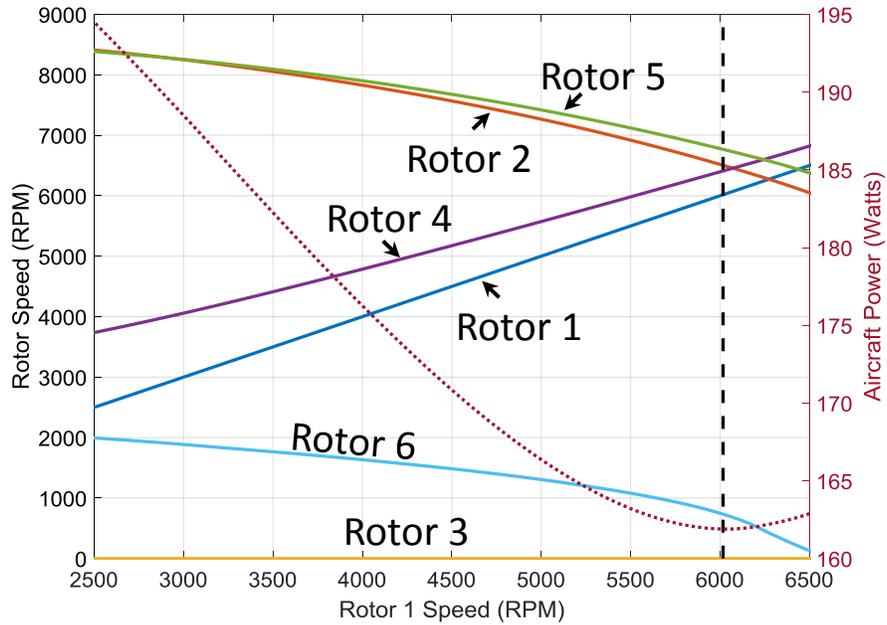


Figure 13: Trimmed Rotor Speeds for Rotor 3 Failure in Forward Flight - Edge First

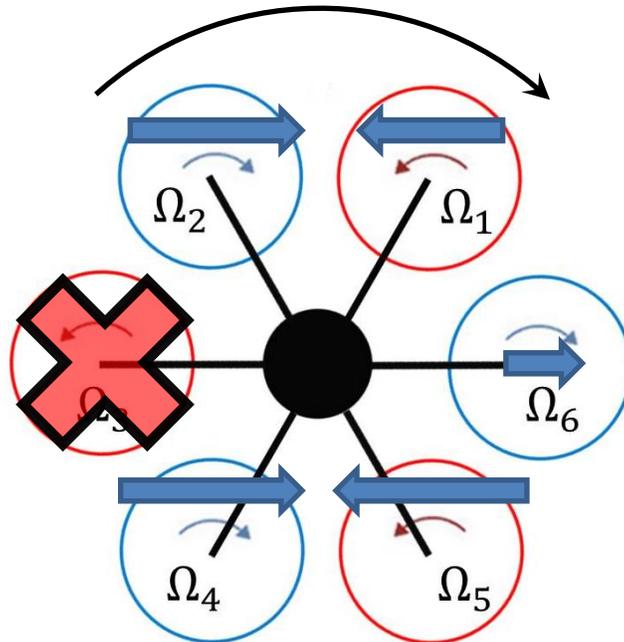


Figure 14: Rotor Side Forces, Rotor 3 Failure in Forward Flight

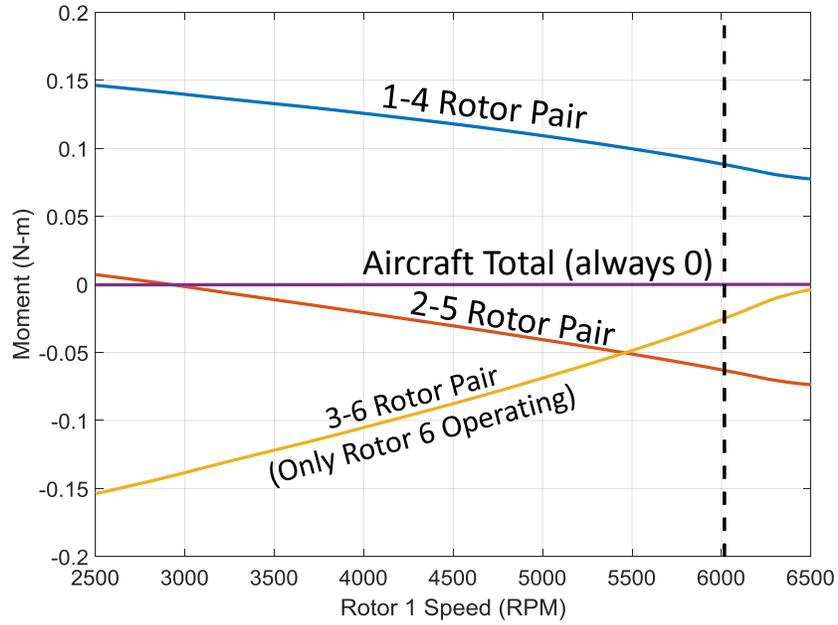


Figure 15: Rolling Moment Contributions by Rotor Pair, Rotor 3 Failure in Forward Flight

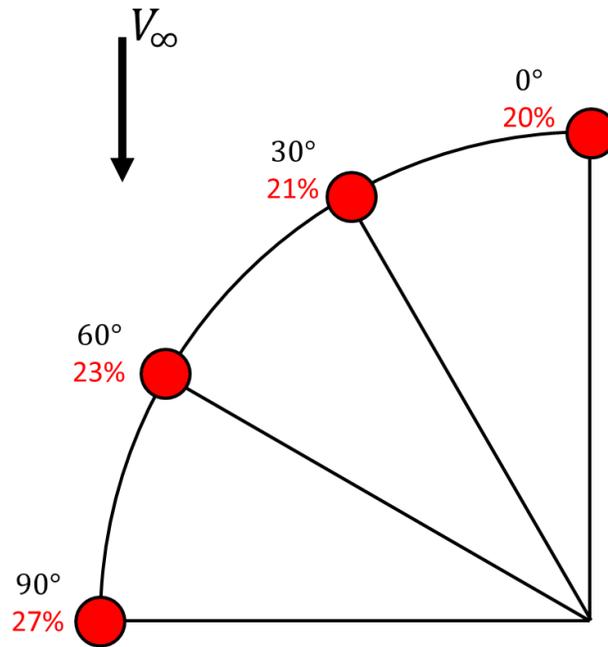


Figure 16: Power Penalty in Forward Flight by Rotor Failure Location